

Linear Buckling Analysis of a Truss Tower with Dead Loads

Introduction

Trusses are commonly used to create light structures that can support heavy loads. When designing such a structure, it is important to ensure its safety. For a tower made of bars, buckling can cause the structure to collapse due to compressive loads that come from external vertical loading and from the self weight of the structure. Linear buckling analysis is a common way to calculate the critical buckling load. The calculated critical load factor is usually interpreted as a common multiplicative factor on all loads. Sometimes, you wants to evaluate the critical value for a specific load, keeping some other loads constant. Distinguishing between constant (dead) loads and varying (live) loads makes it possible to get the critical buckling load for only certain loads.

This example shows how to perform a linear bucking analysis with dead loads on a guyed truss tower where the guy pretension and the gravity force are the dead loads, and the vertical force at the top of the tower is the live load.

Model Definition

The model geometry consists of a 19 m tall truss tower with a rectangular section. The critical buckling load is computed using the linear buckling analysis available in the Truss interface.

The geometry is the periodic structure represented in Figure 1. It consists of 19 blocks of trusses. Each block has a width of 0.45 m, a depth of 0.40 m and a height of 1.0 m. The trusses that are perpendicular to the ground are thicker and have an outer radius of 15 mm and an inner radius of 10 mm. The remaining trusses have an outer radius of 10 mm and an inner radius of 7 mm. The tower is made out of structural steel, which is one of the predefined materials in the material library. Two sets of four guys are fixed at each corner of the tower, at two different height. The tower and guys are fixed at the ground level, the guys are prestressed at 4 kN, and gravity is applied on the tower. A vertical load is applied at the four top points, one fourth of the unit load each. Hence the critical load factor returned by the linear buckling analysis corresponds to the external load that would cause the collapse of the structure.

The guys are modeled using the Wire interface. As long as the guys are in tension, they act as bars, so it would also have been possible to model them in the Truss interface.

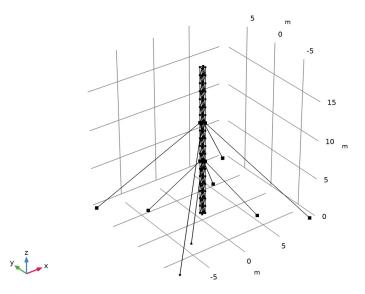


Figure 1: Geometry of the guyed truss tower.

In linear buckling theory the stiffness matrix K is separated into a linear part and a linearized nonlinear part which is proportional to the applied load

$$K = K_L + \lambda K_{NL}(P_0)$$

In the presence of dead loads, the loads are separated between constant dead loads Q_0 and live loads P_0

$$K = K_L + K_{NL}(Q_0) + \lambda K_{NL}(P_0)$$

For a better linearization of nonlinear problems, the live loads are not calculated alone but associated with the dead loads. Hence, an initial stationary solution is calculated with dead loads only. Then, a second solution is calculated with dead and live loads. The stiffness matrix is then

$$K = K_L + K_{NL}(Q_0) + \lambda (K_{NL}(Q_0 \cup P_0) - K_{NL}(Q_0))$$

$$K = K_L + (1 - \lambda)K_{NL}(Q_0) + \lambda K_{NL}(Q_0 \cup P_0)$$

These two solutions are then selected as the linearization point and live load solution in the linear buckling study step.

In addition to the buckling of the whole structure, the safety against buckling of individual truss members must be studied. The critical compressive load for an individual bar can be calculated from material properties and the geometry of the bar:

$$F_{c,\,\mathrm{bar}} = \frac{\pi^2 E I_{\mathrm{min}}}{\left(K_{\mathrm{bar}} L_{\mathrm{bar}}\right)^2}$$

Here, I_{\min} is the moment of inertia in the weakest direction, based on the cross section data, and $L_{\rm bar}$ is the length of the bar. In this case the effective length factor $K_{\rm bar}$ is kept to its default value of 1, which corresponds to a pinned-pinned configuration. From this critical force and the compressive axial force N, the failure index f_i and the local buckling safety factor s_f can be calculated:

$$f_{i} = \frac{-N}{F_{c, \text{bar}}}$$

$$s_{\mathbf{f}} = \frac{1}{f_{\mathbf{i}}}$$

In presence of dead loads, the aim is to find the multiplicative factor to live loads which is needed to reach the critical force:

$$-N(Q_0) + \lambda(-N(Q_0 \cup P_0) + N(Q_0)) = F_{c, \text{ bar}}$$

$$f_i(Q_0) + \lambda(f_i(Q_0 \cup P_0) - f_i(Q_0)) = 1$$

So the safety factor of the live load can be calculated from the live and dead load solutions:

$$s_{\mathbf{f}}(P_0) = \frac{1 - f_{\mathbf{i}}(Q_0)}{f_{\mathbf{i}}(Q_0 \cup P_0) - f_{\mathbf{i}}(Q_0)}$$

Results and Discussion

The two first buckling modes, corresponding respectively to buckling in the depth direction (y direction) and width direction, are shown in Figure 2 and Figure 3. The prestressed guys make it possible to increase the critical load factor up to 92 kN and 117 kN, respectively.

Critical load factor=92196 Line: Displacement magnitude (m) Line: Displacement magnitude (m)

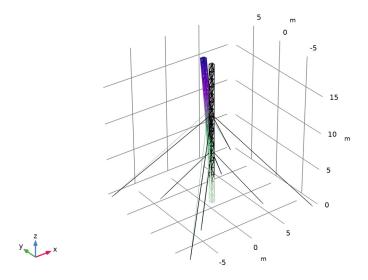


Figure 2: First buckling mode.

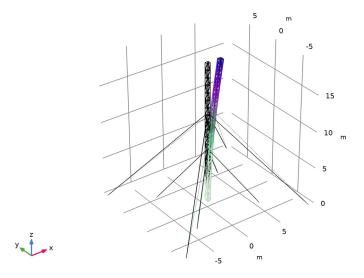


Figure 3: Second buckling mode.

Figure 4 plots the local buckling index for live loads in the tower. The calculated minimal value for safety factor is 131 kN for the diagonal bars at the base of the tower.

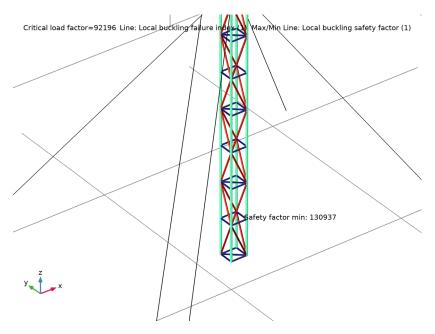


Figure 4: Maximum local buckling index, with value of minimal safety factor.

Notes About the COMSOL Implementation

The Linear Buckling study step has a section for setting the linearization point and the live load solutions. In the absence of dead loads, only one solution is needed, so the live load option must be kept at the default setting Same as linearization point. When dead loads are present, two different solutions must be selected. They can come, for example, from two different study steps, from two different parameter values of an auxiliary sweep, or from two load cases. In this model, two stationary study steps are used, the first one with live load disabled.

Application Library path: Structural Mechanics Module/ Buckling_and_Wrinkling/truss_tower_buckling_guys

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **1** 3D.
- 2 In the Select Physics tree, select Structural Mechanics>Truss (truss).
- 3 Click Add.
- 4 In the Select Physics tree, select Structural Mechanics>Wire (wire).
- 5 Click Add.
- 6 Click Study.
- 7 In the Select Study tree, select Preset Studies for Selected Physics Interfaces> Linear Buckling.
- 8 Click **Done**.

GLOBAL DEFINITIONS

Tower Geometric Parameters

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, type Tower Geometric Parameters in the Label text field.
- 3 Locate the Parameters section. Click Load from File.
- 4 Browse to the model's Application Libraries folder and double-click the file truss_tower_buckling_geometric_parameters.txt.

Guy Geometric Parameters

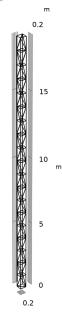
- I In the Home toolbar, click Pi Parameters and choose Add>Parameters.
- 2 In the Settings window for Parameters, type Guy Geometric Parameters in the Label text field.
- **3** Locate the **Parameters** section. In the table, enter the following settings:

Name	Expression	Value	Description
gnd_attach_gu y1	6[m]	6 m	Ground attachment distance of guy 1
gnd_attach_gu y2	12[m]	12 m	Ground attachment distance of guy 2
guy_d	5[mm]	0.005 m	Diameter of guy 1 and guy 2

Name	Expression	Value	Description
guy_height1	7[m]	7 m	Height of guy 1
guy_height2	12[m]	12 m	Height of guy 2
guy_area	pi/4*guy_d^2	1.9635E-5 m ²	Area of guy 1 and guy 2
kA	3.927e6[N]	3.927E6 N	Axial stiffness
rhoL	0.15413[kg/ m]	0.15413 kg/m	Mass per unit length

GEOMETRY I

- I In the Geometry toolbar, click Insert Sequence and choose Insert Sequence.
- **2** Browse to the model's Application Libraries folder and double-click the file truss_tower_buckling.mph.
- 3 In the Geometry toolbar, click Build All.





Array I (arr1), Array 2 (arr2), Block I (blk1), Convert to Curve I (ccur1), Line Segment I (ls1), Line Segment 2 (ls2), Mirror I (mir1), Polygon I (pol1)

- I In the Model Builder window, under Component I (compl)>Geometry I, Ctrl-click to select Block I (blkl), Polygon I (poll), Line Segment I (lsl), Line Segment 2 (ls2), Convert to Curve I (ccurl), Mirror I (mirl), Array I (arrl), and Array 2 (arr2).
- 2 Right-click and choose Group.

Tower

In the **Settings** window for **Group**, type Tower in the **Label** text field.

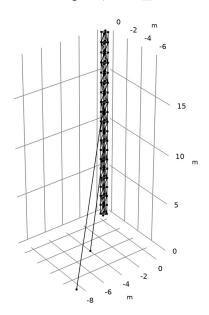
Line Segment 3 (Is3)

- In the Geometry toolbar, click \bigoplus More Primitives and choose Line Segment.
- 2 In the Settings window for Line Segment, locate the Starting Point section.
- **3** From the **Specify** list, choose **Coordinates**.
- 4 In the z text field, type guy height1-mod(guy height1, height).
- **5** Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 6 In the x text field, type -gnd attach guy1*width/sqrt(width^2+depth^2).
- 7 In the y text field, type -gnd_attach_guy1*depth/sqrt(width^2+depth^2).
- 8 Locate the Selections of Resulting Entities section. Find the Cumulative selection subsection. Click New.
- 9 In the New Cumulative Selection dialog box, type Guy 1 in the Name text field.
- IO Click OK.
- II In the Settings window for Line Segment, click **Build All Objects**.

Line Segment 4 (Is4)

- In the Geometry toolbar, click \bigoplus More Primitives and choose Line Segment.
- 2 In the Settings window for Line Segment, locate the Starting Point section.
- 3 From the Specify list, choose Coordinates.
- 4 In the z text field, type guy height2-mod(guy height2, height).
- **5** Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 6 In the x text field, type -gnd attach guy2*width/sqrt(width^2+depth^2).
- 7 In the y text field, type -gnd_attach_guy2*depth/sqrt(width^2+depth^2).
- 8 Locate the Selections of Resulting Entities section. Find the Cumulative selection subsection. Click New.
- 9 In the New Cumulative Selection dialog box, type Guy 2 in the Name text field.
- IO Click OK.

II In the Settings window for Line Segment, click | Build All Objects.





Mirror 2 (mir2)

- I In the Geometry toolbar, click Transforms and choose Mirror.
- 2 Select the objects Is3 and Is4 only.
- ${f 3}$ In the Settings window for Mirror, locate the Input section.
- 4 Select the Keep input objects check box.
- 5 Locate the Point on Plane of Reflection section. In the x text field, type width/2.
- $\boldsymbol{6}$ Locate the Normal Vector to Plane of Reflection section. In the \boldsymbol{x} text field, type 1.
- 7 In the z text field, type 0.
- 8 Click **Build All Objects**.

Mirror 3 (mir3)

- I In the Geometry toolbar, click Transforms and choose Mirror.
- 2 Select the objects Is3, Is4, mir2(1), and mir2(2) only.
- 3 In the Settings window for Mirror, locate the Input section.
- 4 Select the Keep input objects check box.
- 5 Locate the Point on Plane of Reflection section. In the y text field, type depth/2.

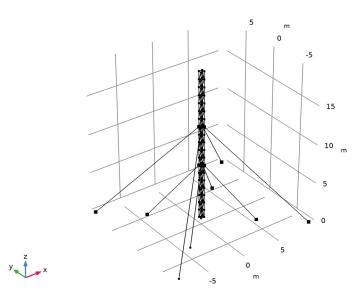
- 6 Locate the Normal Vector to Plane of Reflection section. In the y text field, type 1.
- 7 In the z text field, type 0.
- 8 Click Build All Objects.
- **9** Click the **Go to Default View** button in the **Graphics** toolbar.

Line Segment 3 (Is3), Line Segment 4 (Is4), Mirror 2 (mir2), Mirror 3 (mir3)

- I In the Model Builder window, under Component I (compl)>Geometry I, Ctrl-click to select Line Segment 3 (ls3), Line Segment 4 (ls4), Mirror 2 (mir2), and Mirror 3 (mir3).
- 2 Right-click and choose Group.

Guys

I In the Settings window for Group, type Guys in the Label text field.

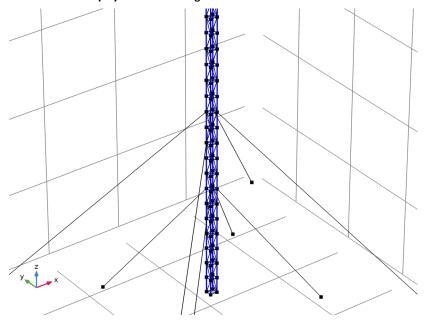


DEFINITIONS

Vertical Edges

- I In the **Definitions** toolbar, click **\(\frac{1}{2} \) Explicit**.
- 2 In the Settings window for Explicit, type Vertical Edges in the Label text field.
- 3 Locate the Input Entities section. From the Geometric entity level list, choose Edge.
- **4** Select Edges 5, 112, 180, and 240 only.

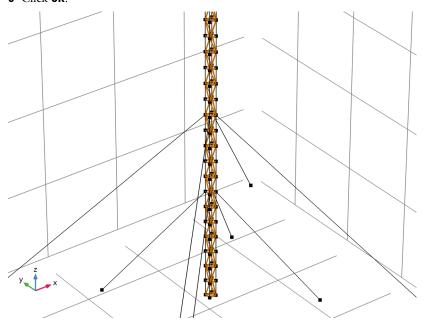
5 Select the Group by continuous tangent check box.



Transversal Truss Edges

- 2 In the Settings window for Complement, type Transversal Truss Edges in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Edge.
- 4 Locate the Input Entities section. Under Selections to invert, click + Add.
- 5 In the Add dialog box, in the Selections to invert list, choose Vertical Edges, Guy I, and Guy 2.

6 Click OK.



Tower

- I In the **Definitions** toolbar, click **Union**.
- 2 In the Settings window for Union, type Tower in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Edge.
- 4 Locate the Input Entities section. Under Selections to add, click + Add.
- 5 In the Add dialog box, in the Selections to add list, choose Vertical Edges and Transversal Truss Edges.
- 6 Click OK.

Guys

- I In the **Definitions** toolbar, click **Union**.
- 2 In the Settings window for Union, type Guys in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Edge.
- 4 Locate the Input Entities section. Under Selections to add, click + Add.
- 5 In the Add dialog box, in the Selections to add list, choose Guy I and Guy 2.
- 6 Click OK.

TRUSS (TRUSS)

- I In the Model Builder window, under Component I (compl) click Truss (truss).
- 2 In the Settings window for Truss, locate the Edge Selection section.
- **3** From the **Selection** list, choose **Tower**.

Cross-Section Data (Vertical Edges)

- I In the Model Builder window, under Component I (compl)>Truss (truss) click Cross-Section Data I.
- 2 In the Settings window for Cross-Section Data, type Cross-Section Data (Vertical Edges) in the Label text field.
- 3 Locate the Cross-Section Definition section. From the list, choose Common sections.
- 4 From the Section type list, choose Pipe.
- **5** In the d_o text field, type do1.
- **6** In the d_i text field, type di1.

Cross-Section Data (Other Truss Edges)

- I In the Physics toolbar, click **Edges** and choose Cross-Section Data.
- 2 In the Settings window for Cross-Section Data, locate the Edge Selection section.
- 3 From the Selection list, choose Transversal Truss Edges.
- 4 Locate the Cross-Section Definition section. From the list, choose Common sections.
- 5 From the Section type list, choose Pipe.
- **6** In the d_o text field, type do2.
- **7** In the d_i text field, type di2.
- 8 In the Label text field, type Cross-Section Data (Other Truss Edges).

Pinned I

- I In the Physics toolbar, click Points and choose Pinned.
- **2** Select Points 1–5, 25, 45, 65, and 85–88 only.

Gravity I

In the Physics toolbar, click A Global and choose Gravity.

Point Load 1

- I In the Physics toolbar, click Points and choose Point Load.
- **2** Select Points 24, 44, 64, and 84 only.
- 3 In the Settings window for Point Load, locate the Force section.

4 Specify the $\mathbf{F}_{\mathbf{P}}$ vector as

0	x
0	у
-1[N]/4	z

Since

WIRE (WIRE)

- I In the Model Builder window, under Component I (compl) click Wire (wire).
- 2 In the Settings window for Wire, locate the Edge Selection section.
- **3** From the **Selection** list, choose **Guys**.

Use the same dependent variable in both physics interfaces in order to share the displacement where the guys are attached to the tower.

4 Click to expand the Dependent Variables section. In the Displacement field text field, type u.

Elastic Wire I

- I In the Model Builder window, under Component I (compl)>Wire (wire) click Elastic Wire I.
- 2 In the Settings window for Elastic Wire, locate the Basic Section Properties section.
- **3** In the A text field, type guy area.

Initial Stress and Strain I

- I In the Physics toolbar, click 🖳 Attributes and choose Initial Stress and Strain.
- 2 In the Settings window for Initial Stress and Strain, locate the Edge Selection section.
- 3 From the Selection list, choose Guys.
- 4 Locate the Initial Stress and Strain section. In the N_i text field, type 4[kN].

Pinned I

- I In the Physics toolbar, click Points and choose Pinned.
- 2 Select Points 1-4 and 85-88 only.

In the Physics toolbar, click A Global and choose Gravity.

ADD MATERIAL

I In the Home toolbar, click **Add Material** to open the Add Material window.

- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Structural steel.
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click **‡ Add Material** to close the **Add Material** window.

MATERIALS

Structural steel (mat I)

- I In the Settings window for Material, locate the Geometric Entity Selection section.
- 2 From the Selection list, choose Tower.

Wire material

- I In the Model Builder window, right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Wire material in the Label text field.
- 3 Locate the Geometric Entity Selection section. From the Selection list, choose Guys.
- **4** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Axial stiffness	k_A	kA	N	Elastic wire
Mass per unit length	rho_L	rhoL	kg/m	Elastic wire

MESH I

Edge I

- I In the Mesh toolbar, click A Boundary and choose Edge.
- 2 In the Settings window for Edge, locate the Edge Selection section.
- 3 From the Selection list, choose All edges.

Distribution I

- I Right-click Edge I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Edge Selection section.
- 3 From the Selection list, choose All edges.
- 4 Locate the Distribution section. In the Number of elements text field, type 1.

Distribution 2

- I In the Model Builder window, right-click Edge I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Edge Selection section.

- 3 From the Selection list, choose Guy 1.
- 4 Locate the **Distribution** section. In the **Number of elements** text field, type 10.

Distribution 3

- I Right-click **Edge I** and choose **Distribution**.
- 2 In the Settings window for Distribution, locate the Edge Selection section.
- 3 From the Selection list, choose Guy 2.
- 4 Locate the **Distribution** section. In the **Number of elements** text field, type 10.
- 5 Click III Build All.

The buckling study requires two stationary study steps to compute live and dead load solutions.

STUDY I

Stationary 2

- I In the Study toolbar, click Study Steps and choose Stationary>Stationary.
- 2 Right-click Study I>Step 3: Stationary 2 and choose Move Up.

Use the first stationary study step to solve for the dead loads only.

Stationary, Dead Loads Only

- I In the Model Builder window, click Step I: Stationary.
- 2 In the Settings window for Stationary, type Stationary, Dead Loads Only in the Label text field.
- 3 Locate the Physics and Variables Selection section. Select the Modify model configuration for study step check box.
- 4 In the tree, select Component I (compl)>Truss (truss)>s.
- 5 Click / Disable.

In the second stationary study step, both live and dead loads are applied.

Stationary, Dead + Live Loads

- I In the Model Builder window, under Study I click Step 2: Stationary 2.
- 2 In the Settings window for Stationary, type Stationary, Dead + Live Loads in the Label text field.

You need to generate the solver sequence to enable the selection of solutions in the linear buckling steps.

Solution I (soll)

In the Study toolbar, click Show Default Solver.

Step 3: Linear Buckling

- I In the Model Builder window, under Study I click Step 3: Linear Buckling.
- 2 In the Settings window for Linear Buckling, locate the Study Settings section.
- 3 In the Desired number of buckling modes text field, type 2.
- 4 Find the Values of linearization point subsection. From the Settings list, choose User controlled.
- 5 From the Use list, choose Solution Store I (sol2).
- 6 Find the Live loads solution subsection. From the Settings list, choose User controlled.
- 7 In the Study toolbar, click **Compute**.

RESULTS

line l

- I In the Model Builder window, expand the Mode Shape (truss) node, then click Line I.
- 2 In the Settings window for Line, locate the Coloring and Style section.
- 3 In the Radius scale factor text field, type 6.

Line 1

- I In the Model Builder window, expand the Results>Mode Shape (wire) node.
- 2 Right-click Line I and choose Copy.

Line 2

- I In the Model Builder window, right-click Mode Shape (truss) and choose Paste Line.
- 2 In the Settings window for Line, click to expand the Inherit Style section.
- 3 From the Plot list, choose Line 1.

Mode Shape (truss)

- I In the Model Builder window, click Mode Shape (truss).
- 2 In the Mode Shape (truss) toolbar, click Plot.

Add variables to assess the risk of local buckling.

DEFINITIONS

Variables 1

- I In the Model Builder window, under Component I (compl) right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
lbs_f	<pre>(1-withsol('sol2', truss.lbf_i))/ (withsol('sol3', truss.lbf_i)- withsol('sol2', truss.lbf_i))</pre>		Local buckling safety factor
lbf_i	1/lbs_f		Local buckling failure index

STUDY I

In the Study toolbar, click C Update Solution.

RESULTS

Local Buckling

- I In the Home toolbar, click Add Plot Group and choose 3D Plot Group.
- 2 In the Settings window for 3D Plot Group, type Local Buckling in the Label text field.

line l

- I Right-click Local Buckling and choose Line.
- 2 In the Settings window for Line, locate the Expression section.
- 3 In the Expression text field, type 1bf i.
- **4** Locate the **Coloring and Style** section. Clear the **Color legend** check box.
- **5** From the **Line type** list, choose **Tube**.
- 6 In the Tube radius expression text field, type truss.re.
- 7 Select the Radius scale factor check box. In the associated text field, type 2.
- 8 Click to expand the Quality section. From the Smoothing list, choose None.
- **9** In the Local Buckling toolbar, click **Plot**.

Local Buckling

In the Model Builder window, click Local Buckling.

Max/Min Line I

- I In the Local Buckling toolbar, click More Plots and choose Max/Min Line.
- 2 In the Settings window for Max/Min Line, locate the Expression section.
- 3 In the Expression text field, type lbs_f.
- 4 Locate the Display section. From the Display list, choose Min.
- **5** Locate the **Text Format** section. In the **Prefix** text field, type Safety factor .
- 6 In the Local Buckling toolbar, click Plot.