

# Large Deformation Analysis of a Beam

In this example you study the deflection of a cantilever beam undergoing very large deflections. The model is called "Straight Cantilever GNL Benchmark" and is described in detail in section 5.2 of NAFEMS Background to Finite Element Analysis of Geometric Non-linearity Benchmarks (Ref. 1). A schematic description of the beam and its characteristics is shown in Figure 1.

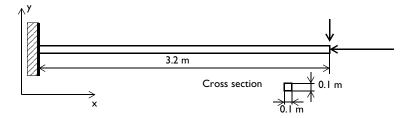


Figure 1: Cantilever beam geometry.

#### GEOMETRY

- The length of the beam is 3.2 m.
- The cross section is a square with side lengths 0.1 m.

#### MATERIAL

The beam is linear elastic with  $E = 2.1 \cdot 10^{11} \text{ N/m}^2$  and v = 0.

#### CONSTRAINTS AND LOADS

- The left end is fixed.
- The right end is subjected to a total load of  $F_r = -3.844 \cdot 10^6 \text{ N}$  and  $F_v = -3.844 \cdot 10^3 \text{ N}$ .

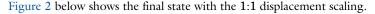
#### MODELING IN COMSOL

This problem is modeled separately using both Solid Mechanics and Beam interfaces and the results are compared with the benchmark value. Using the Solid Mechanics interface, the problem is modeled as a "plane stress" problem considering that out-of-plane dimension is small. Poisson's ratio v is set to zero to make the boundary conditions consistent with the beam theory assumptions. The load on the right end of the beam is modeled as a uniformly distributed boundary load, corresponding to the specified total load.

In the second part of this problem, a linear buckling analysis study is carried out to compute the critical buckling load of the structure.

## Results and Discussion

Due to the large compressive axial load and the slender geometry, this is a buckling problem. If you are to study the buckling and post-buckling behavior of a symmetric problem, it is necessary to perturb the symmetry somewhat. Here the small transversal load serves this purpose. An alternative approach would be to introduce an initial imperfection in the geometry.



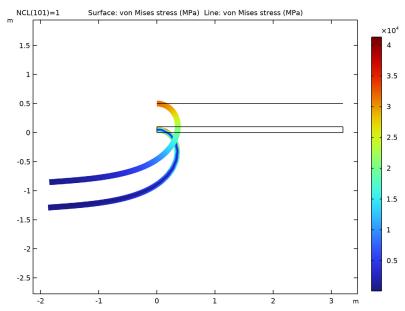


Figure 2: The effective von Mises stress of the deformed beam.

The horizontal and vertical displacements of the tip versus the compressive load normalized by its maximum value are shown in Figure 3.

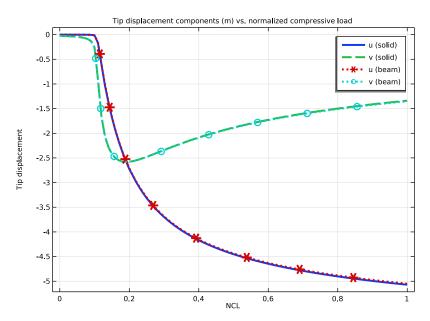


Figure 3: Horizontal and vertical tip displacements versus normalized compressive load.

Table 1 contains a summary of some significant results. Because the reference values are given as graphs, an estimate of the error caused by reading this graph is added:

TABLE I: COMPARISON BETWEEN MODEL RESULTS AND REFERENCE VALUES.

QUANTITY	COMSOL (SOLID)	COMSOL (BEAM)	REFERENCE
Maximum vertical displacement at the tip	-2.58	-2.58	-2.58 ± 0.02
Final vertical displacement at the tip	-1.34	-1.35	-1.36 ± 0.02
Final horizontal displacement at the tip	-5.07	-5.05	-5.04 ± 0.04

The results are in excellent agreement, especially considering the coarse mesh used.

The plot of the axial deflection reveals that an instability occurs at a parameter value close to 0.1, corresponding to the compressive load  $3.84 \cdot 10^5$  N. It is often seen in practice that the critical load of an imperfect structure is significantly lower than that of the ideal structure.

This problem (without the small transverse load) is usually referred to as the Euler-1 case. The theoretical critical load is

$$P_{\rm c} = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 \cdot 2.1 \cdot 10^{11} \cdot (0.1^4 / 12)}{4 \cdot 3.2^2} = 4.22 \cdot 10^5 \text{ N}$$

Figure 4 shows the first buckling mode of the beam computed from a linear buckling analysis.

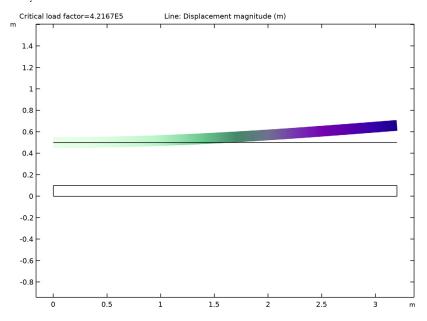


Figure 4: First buckling mode of the beam.

# Reference

1. A.A. Becker, Background to Finite Element Analysis of Geometric Non-linearity Benchmarks, NAFEMS, Ref: -R0065, Glasgow, 1999.

**Application Library path:** Structural\_Mechanics\_Module/ Verification\_Examples/large\_deformation\_beam From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

#### MODEL WIZARD

- I In the Model Wizard window, click **2D**.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.
- 4 In the Select Physics tree, select Structural Mechanics>Beam (beam).
- 5 Click Add.
- 6 Click 🔵 Study.
- 7 In the Select Study tree, select General Studies>Stationary.
- 8 Click **Done**.

#### **GLOBAL DEFINITIONS**

Define parameters for the geometric data, compressive and transverse load components as well as a parameter that you will use to gradually turn up the compressive load.

#### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file large\_deformation\_beam\_parameters.txt.

By restricting the range of parameter **NCL** to [0, 1], it serves as a compressive load normalized by the maximum compressive load.

#### **GEOMETRY I**

#### Rectangle I (rI)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 1.
- 4 In the Height text field, type d.

Polygon I (poll)

- I In the Geometry toolbar, click / Polygon.
- 2 In the Settings window for Polygon, locate the Coordinates section.
- **3** In the table, enter the following settings:

x (m)	y (m)
0	5*d
1	5*d

4 Click Build All Objects.

Form Union (fin)

- I In the Model Builder window, click Form Union (fin).
- 2 In the Settings window for Form Union/Assembly, click | Build Selected.

#### **GLOBAL DEFINITIONS**

In this example, the same material data will be referenced for Solid Mechanics and Beam interfaces, hence it can be added as a Global Material in the model. Using Material Link node, we assign the Global Material to different domains, boundaries and edges of the structure.

Material I (mat I)

- I In the Model Builder window, under Global Definitions right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, click to expand the Material Properties section.
- 3 In the Material properties tree, select Basic Properties>Density.
- 4 Click + Add to Material.
- 5 In the Material properties tree, select Basic Properties>Poisson's Ratio.
- 6 Click + Add to Material.
- 7 In the Material properties tree, select Basic Properties>Young's Modulus.
- 8 Click + Add to Material.
- **9** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	7850	kg/m³	Basic
Poisson's ratio	nu	0	ı	Basic
Young's modulus	E	2.1e5[MPa]	Pa	Basic

#### MATERIALS

Material Link I (matlnk I)

In the Model Builder window, under Component I (compl) right-click Materials and choose More Materials>Material Link.

Material Link 2 (matlnk2)

- I Right-click Materials and choose More Materials>Material Link.
- 2 In the Settings window for Material Link, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 Select Boundary 4 only.

Add physics settings for the Solid Mechanics interface.

#### SOLID MECHANICS (SOLID)

- I In the Model Builder window, under Component I (compl) click Solid Mechanics (solid).
- 2 In the Settings window for Solid Mechanics, locate the 2D Approximation section.
- 3 From the list, choose Plane stress.
- **4** Locate the **Thickness** section. In the d text field, type d.

#### Fixed Constraint I

- I In the Physics toolbar, click Boundaries and choose Fixed Constraint.
- 2 Select Boundary 1 only.

#### Boundary Load 1

- I In the Physics toolbar, click Boundaries and choose Boundary Load.
- 2 Select Boundary 5 only.
- 3 In the Settings window for Boundary Load, locate the Force section.
- 4 From the Load type list, choose Total force.
- **5** Specify the  $\mathbf{F}_{\text{tot}}$  vector as

NCL*F Lx	x
F_Ly	у

#### BEAM (BEAM)

- I In the Model Builder window, under Component I (compl) click Beam (beam).
- 2 In the Settings window for Beam, locate the Boundary Selection section.
- 3 Click Clear Selection.

4 Select Boundary 4 only.

Cross-Section Data 1

- I In the Model Builder window, under Component I (compl)>Beam (beam) click Cross-Section Data 1.
- 2 In the Settings window for Cross-Section Data, locate the Cross-Section Definition section.
- 3 From the list, choose Common sections.
- **4** In the  $h_v$  text field, type d.
- **5** In the  $h_z$  text field, type d.

Fixed Constraint I

- I In the Physics toolbar, click Points and choose Fixed Constraint.
- 2 Select Point 3 only.

Point Load 1

- I In the Physics toolbar, click Points and choose Point Load.
- 2 In the Settings window for Point Load, locate the Force section.
- **3** Specify the  $\mathbf{F}_{\mathbf{P}}$  vector as

NCL*F_Lx	x
F_Ly	у

**4** Select Point 6 only.

Add another unit point load for the linear buckling analysis.

Point Load 2

- I Right-click Point Load I and choose Duplicate.
- 2 In the Settings window for Point Load, locate the Force section.
- **3** Specify the  $\mathbf{F}_{\mathbf{P}}$  vector as

- 1	x
0	у

#### MESH I

Edge I

- I In the Mesh toolbar, click A Edge.
- 2 Select Boundaries 2-4 only.

#### Distribution 1

- I Right-click **Edge I** and choose **Distribution**.
- 2 In the Settings window for Distribution, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.
- 4 Select Boundary 4 only.
- **5** Locate the **Distribution** section. In the **Number of elements** text field, type 40.

#### Distribution 2

- I In the Model Builder window, right-click Edge I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.
- 4 Select Boundaries 2 and 3 only.
- 5 Locate the Distribution section. In the Number of elements text field, type 20.

#### Mapped I

- I In the Mesh toolbar, click Mapped.
- 2 In the Settings window for Mapped, click **Build All**.

#### STUDY I

#### Step 1: Stationary

- I In the Model Builder window, under Study I click Step I: Stationary.
- 2 In the Settings window for Stationary, locate the Study Settings section.
- 3 Select the Include geometric nonlinearity check box.
- 4 Locate the Physics and Variables Selection section. Select the Modify model configuration for study step check box.
- 5 In the tree, select Component I (compl)>Beam (beam)>Point Load 2.
- 6 Click O Disable.
- 7 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** check box.
- 8 Click + Add.
- **9** In the table, enter the following settings:

Parameter name	Parameter value list	
NCL (Normalized compressive load)	range(0,0.01,1)	

10 Right-click Study I>Step I: Stationary and choose Get Initial Value for Step.

#### STUDY I

Solver Configurations

In the Model Builder window, expand the Study I>Solver Configurations node.

Solution I (soll)

- I In the Model Builder window, expand the Study I>Solver Configurations>Solution I (soll) node, then click Stationary Solver I.
- 2 In the Settings window for Stationary Solver, locate the General section.
- 3 In the Relative tolerance text field, type 1e-4.
- 4 In the Model Builder window, expand the Study I>Solver Configurations> Solution I (soll)>Stationary Solver I node.
- 5 Right-click Stationary Solver I and choose Segregated.
- 6 In the Settings window for Segregated, locate the General section.
- 7 From the Termination technique list, choose Iterations.
- 8 In the Model Builder window, expand the Study I>Solver Configurations>
  Solution I (sol1)>Stationary Solver I>Segregated I node, then click Segregated Step.
- 9 In the Settings window for Segregated Step, locate the General section.
- 10 In the Variables list, select
  Displacement field (material and geometry frames) (compl.beam.uLin).
- II Under Variables, click Delete.
- 12 Under Variables, click Delete.
- **13** Click to expand the **Method and Termination** section. From the **Termination technique** list, choose **Tolerance**.
- 14 In the Model Builder window, right-click Segregated 1 and choose Segregated Step.
- 15 In the Settings window for Segregated Step, locate the General section.
- 16 Under Variables, click + Add.
- 17 In the Add dialog box, in the Variables list, choose Rotation field (material and geometry frames) (compl.beam.thLin) and Displacement field (material and geometry frames) (compl.beam.uLin).
- 18 Click OK.
- 19 In the Settings window for Segregated Step, locate the Method and Termination section.
- 20 From the Nonlinear method list, choose Automatic (Newton).
- 21 In the Maximum number of iterations text field, type 200.

**22** In the **Tolerance factor** text field, type 1.

#### Step 1: Stationary

- I In the Model Builder window, under Study I click Step I: Stationary.
- 2 In the Settings window for Stationary, click to expand the Results While Solving section.
- **3** Select the **Plot** check box.
- 4 From the Plot group list, choose Stress (beam).
- 5 In the Home toolbar, click **Compute**.

#### RESULTS

#### Line 1

- I In the Model Builder window, expand the Results>Stress (beam) node, then click Line I.
- 2 In the Settings window for Line, locate the Expression section.
- 3 From the Unit list, choose MPa.
- 4 Right-click Line I and choose Copy.

In the Model Builder window, right-click Stress (solid) and choose Paste Line.

#### Surface I

- I In the Settings window for Surface, locate the Expression section.
- 2 From the Unit list, choose MPa.
- 3 Locate the Coloring and Style section. Click Change Color Table.
- 4 In the Color Table dialog box, select Rainbow>Rainbow in the tree.
- 5 Click OK.

#### line l

- I In the Model Builder window, click Line I.
- 2 In the Settings window for Line, click to expand the Inherit Style section.
- 3 From the Plot list, choose Surface 1.
- 4 Clear the **Tube radius scale factor** check box.

#### Stress (solid and beam)

- I In the Model Builder window, under Results click Stress (solid).
- 2 In the Settings window for 2D Plot Group, type Stress (solid and beam) in the Label text field.

- 3 Locate the Plot Settings section. From the Frame list, choose Material (X, Y, Z).
- 4 In the Stress (solid and beam) toolbar, click on Plot.
- 5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Add a dataset to use for plotting of the results at the tip of the solid beam.

#### Cut Point 2D I

- I In the Results toolbar, click Cut Point 2D.
- 2 In the Settings window for Cut Point 2D, locate the Point Data section.
- 3 In the X text field, type 1.
- 4 In the Y text field, type d/2.
- 5 Click Plot.
- **6** Click the **Zoom Extents** button in the **Graphics** toolbar.

#### Tip Displacement

- I In the Results toolbar, click  $\sim$  ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Tip Displacement in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Cut Point 2D 1.

#### Point Graph 1

- I Right-click Tip Displacement and choose Point Graph.
- 2 In the Settings window for Point Graph, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)> Solid Mechanics>Displacement>Displacement field m>u Displacement field, X-component.
- 3 Click to expand the Coloring and Style section. From the Width list, choose 3.
- 4 Click to expand the **Legends** section. Select the **Show legends** check box.
- 5 From the Legends list, choose Manual.
- **6** In the table, enter the following settings:

# Legends u (solid)

#### Point Graph 2

- I In the Model Builder window, right-click Tip Displacement and choose Point Graph.
- 2 In the Settings window for Point Graph, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>

Solid Mechanics>Displacement>Displacement field - m>v - Displacement field, Ycomponent.

- 3 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose Dashed.
- 4 From the Width list, choose 3.
- **5** Locate the **Legends** section. Select the **Show legends** check box.
- 6 From the Legends list, choose Manual.
- 7 In the table, enter the following settings:

Legends	
٧	(solid)

#### Point Grabh 3

- I Right-click **Tip Displacement** and choose **Point Graph**.
- 2 In the Settings window for Point Graph, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I (soll).
- 4 Locate the **Selection** section. Click to select the **Activate Selection** toggle button.
- **5** Select Point 6 only.
- 6 Click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>Beam>Displacement>Displacement field - m>u2 -Displacement field, X-component.
- 7 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose **Dotted**.
- 8 Find the Line markers subsection. From the Marker list, choose Asterisk.
- **9** From the **Positioning** list, choose **Interpolated**.
- **IO** From the **Width** list, choose **3**.
- II Locate the Legends section. Select the Show legends check box.
- 12 From the Legends list, choose Manual.
- **I3** In the table, enter the following settings:



#### Point Graph 4

I Right-click Point Graph 3 and choose Duplicate.

- 2 In the Settings window for Point Graph, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>Beam> Displacement>Displacement field - m>v2 - Displacement field, Y-component.
- 3 Locate the Coloring and Style section. Find the Line markers subsection. From the Marker list, choose Circle.
- 4 From the Positioning list, choose Interpolated.
- **5** Locate the **Legends** section. In the table, enter the following settings:

# Legends v (beam)

6 In the Tip Displacement toolbar, click **Plot**.

#### Tip Displacement

- I In the Model Builder window, click Tip Displacement.
- 2 In the Settings window for ID Plot Group, click to expand the Title section.
- 3 From the Title type list, choose Manual.
- 4 In the Title text area, type Tip displacement components (m) vs. normalized compressive load.
- 5 Locate the Plot Settings section.
- 6 Select the y-axis label check box. In the associated text field, type Tip displacement.
- 7 In the Tip Displacement toolbar, click **Plot**.
- 8 Click the Zoom Extents button in the Graphics toolbar.

Evaluate the deformation of the structure.

#### Point Evaluation 1

- I In the Results toolbar, click 8.85 Point Evaluation.
- 2 In the Settings window for Point Evaluation, locate the Data section.
- 3 From the Dataset list, choose Cut Point 2D 1.
- 4 From the Parameter selection (NCL) list, choose Last.
- 5 Click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)>Solid Mechanics>Displacement>Displacement field m>u - Displacement field, X-component.

**6** Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
u	m	Solid: x-disp

#### 7 Click **= Evaluate**.

#### Point Evaluation 2

- I Right-click Point Evaluation I and choose Duplicate.
- 2 In the Settings window for Point Evaluation, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I (soll).
- 4 Locate the **Selection** section. Click to select the **Activate Selection** toggle button.
- **5** Select Point 6 only.
- **6** Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
u2	m	Beam: x-disp
uFinal_Ref	m	Reference value for final horizontal displacement at the tip

7 Click ▼ next to **= Evaluate**, then choose **Table I - Point Evaluation I**.

#### Point Evaluation 3

- I In the Model Builder window, under Results>Derived Values right-click Point Evaluation I and choose **Duplicate**.
- 2 In the Settings window for Point Evaluation, locate the Expressions section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
v	m	Solid: y-disp

**4** Click ▼ next to **≡ Evaluate**, then choose **New Table**.

#### Point Evaluation 4

- I In the Model Builder window, under Results>Derived Values right-click Point Evaluation 2 and choose **Duplicate**.
- 2 In the Settings window for Point Evaluation, locate the Expressions section.

**3** In the table, enter the following settings:

Expression	Unit	Description
v2	m	Beam: y-disp
vFinal_Ref	m	Reference value for final vertical displacement at the tip

4 Click ▼ next to **= Evaluate**, then choose **Table 2 - Point Evaluation 3**.

#### Point Evaluation 5

- I In the Model Builder window, under Results>Derived Values right-click Point Evaluation 3 and choose Duplicate.
- 2 In the Settings window for Point Evaluation, locate the Data section.
- 3 From the Parameter selection (NCL) list, choose All.
- **4** Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
abs(v)	m	Solid: y-disp

- 5 Locate the Data Series Operation section. From the Transformation list, choose Maximum.
- 6 Click ▼ next to **= Evaluate**, then choose **New Table**.

#### Point Evaluation 6

- I In the Model Builder window, under Results>Derived Values right-click Point Evaluation 4 and choose Duplicate.
- 2 In the Settings window for Point Evaluation, locate the Data section.
- 3 From the Parameter selection (NCL) list, choose All.
- **4** Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
abs(v2)	m	Beam: y-disp
abs(vMax_Ref)	m	

- 5 Locate the Data Series Operation section. From the Transformation list, choose Maximum.
- 6 Click ▼ next to **= Evaluate**, then choose **Table 3 Point Evaluation 5**.

#### ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.

- 3 Find the Studies subsection. In the Select Study tree, select Preset Studies for Selected Physics Interfaces>Linear Buckling.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

#### STUDY 2

### Step 1: Stationary

- I In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 2 Select the Modify model configuration for study step check box.
- 3 In the tree, select Component I (compl)>Solid Mechanics (solid).
- 4 Click Disable in Model.
- 5 In the tree, select Component I (compl)>Beam (beam)>Point Load I.
- 6 Click Disable.

# Step 2: Linear Buckling

- I In the Model Builder window, click Step 2: Linear Buckling.
- 2 In the Settings window for Linear Buckling, locate the Physics and Variables Selection section.
- 3 Select the Modify model configuration for study step check box.
- 4 In the tree, select Component I (compl)>Solid Mechanics (solid).
- 5 Click Disable in Model.
- 6 In the tree, select Component I (compl)>Beam (beam)>Point Load I.
- 7 Click Disable.
- 8 In the Home toolbar, click **Compute**.

#### RESULTS

Mode Shape (beam)

Click the **Zoom Extents** button in the **Graphics** toolbar.

#### Point Evaluation 7

- I In the Results toolbar, click 8.85 Point Evaluation.
- 2 In the Settings window for Point Evaluation, locate the Data section.
- 3 From the Dataset list, choose Study 2/Solution 2 (sol2).
- 4 Select Point 6 only.

**5** Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description	
For	N	First critical buckling load	

6 Click **= Evaluate**.