



Fast Modeling of a Transmission Line Wilkinson Power Divider

Introduction

Some conventional three-port power dividers are resistive power dividers and T-junction power dividers. Such dividers are either lossy or not matched to the system reference impedance at all ports. In addition, isolation between two coupled ports is not guaranteed. The Wilkinson power divider outperforms the lossless T-junction divider and the resistive divider and does not have the abovementioned issues. This example model simulates a Wilkinson power divider using the Transmission Line physics interface in 2D. This approach is very fast compared to solving Maxwell's equations in 3D. The results present the S-parameters from 1 GHz to 5 GHz and electric potential distribution along the transmission line.

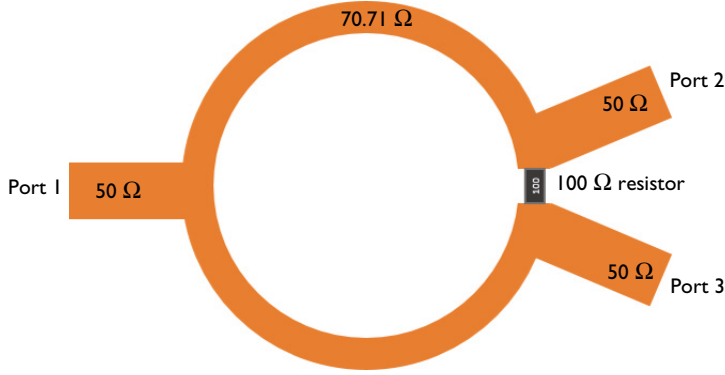


Figure 1: A microstrip Wilkinson power divider is composed of a 100 Ω resistor and transmission lines with two impedance values.

Model Definition

The power divider geometry consists of three straight lines built with Bézier polygons and three curved lines created by circles. Transmission Line Equation features define the distributed parameter values for 50 Ω and 70.71 Ω microstrip lines and it also defines the parameter values for the 100 Ω resistor connecting two 70.71 Ω microstrip lines.

The transmission line parameters for a 50 Ω microstrip line built on a 20 mil substrate with permittivity $\epsilon_r = 3.38$ and 1 oz copper can be calculated accurately from [Ref. 2](#).

TABLE 1: CALCULATED TRANSMISSION LINE PARAMETERS OF A 50 Ω MICROSTRIP LINE.

R	L	G	C
12.41 Ω/m	272.9 nH/m	0 S/m	107.1 pF/m

The contribution of the distributed resistance to the insertion loss with the given substrate properties is less than 0.05 dB. To make the modeling steps simpler in this example, the approximated parameter values in Table 2 are used for a 50 Ω microstrip line.

TABLE 2: SIMPLIFIED TRANSMISSION LINE PARAMETERS OF A 50 Ω MICROSTRIP LINE.

R	L	G	C
0 Ω/m	250 nH/m	0 S/m	100 pF/m

70.71 Ω microstrip line parameters are adjusted using the normalized impedance. The distributed inductance is proportionally scaled and the distributed capacitance is inversely scaled by the normalized impedance of the microstrip line. The distributed resistance for the 100 Ω resistor that connects two 70.71 Ω microstrip lines is scaled inversely by the length of the curvature where the resistor is placed.

In order to have a complete S-parameter report, port sweep option in the physics interface needs to be activated. When the port sweep is combined with a parametric sweep, this will excite ports one by one to generate a full S-parameter matrix plot.

Results and Discussion

In Figure 2 and Figure 3, the electric potential along the transmission lines are plotted. When port 3 is excited, the input electric potential is not delivered to port 2 (Figure 2). Since the device is symmetric with regard to port 2 and 3, the isolation from port 2 to port 3 is also expected when port 2 is excited. This ensures two coupled ports (port 2 and port 3) are isolated from each other. When port 1 is excited, the input electric potential is split equally between port 2 and port 3 (Figure 3).

Figure 4 and Figure 5 show the computed S-parameter plots when port 1 and port 2 is excited, respectively. When the input power is equally split to two coupled ports, the received power level is -3 dB (S_{21} and S_{31}) in each coupled port at 3 GHz. The impedance matching (S_{11} and S_{22}) to the reference characteristic impedance 50 Ω and isolation (S_{23}) are below -50 dB at the center frequency.

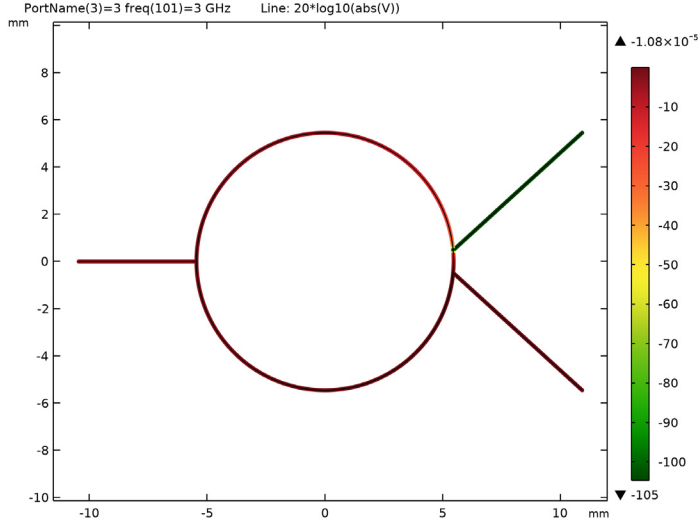


Figure 2: Port 3 is isolated from port 2 when port 3 is excited.

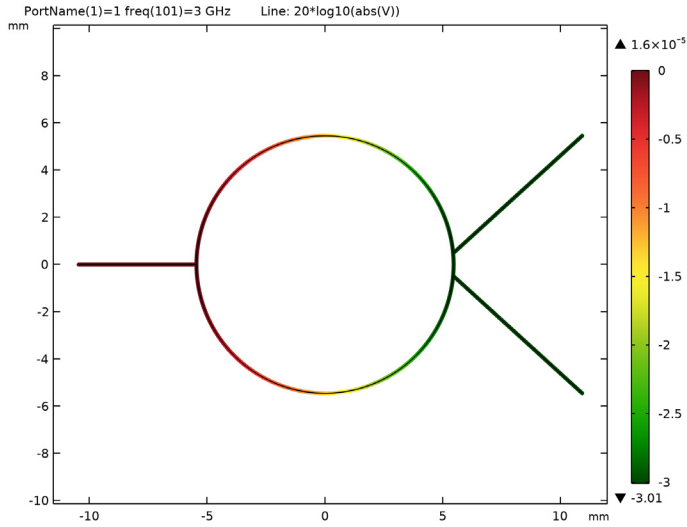


Figure 3: The input voltage is distributed equally between port 2 and port 3 when port 1 is excited.

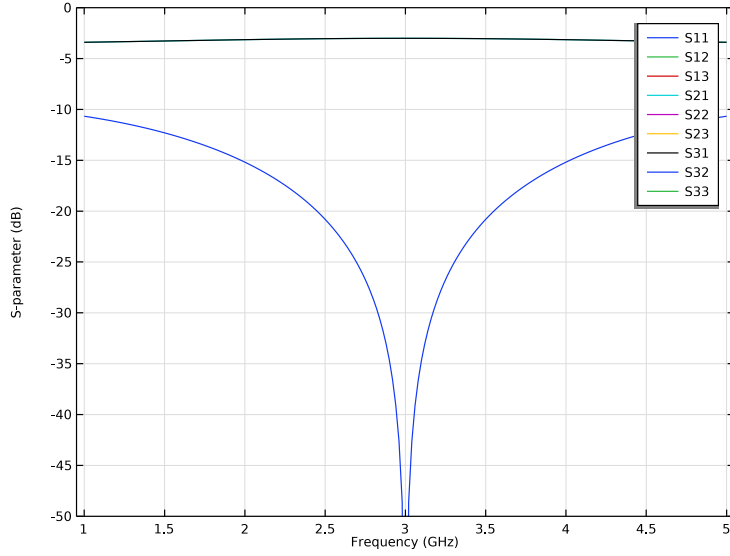


Figure 4: The computed S -parameter plot when port 1 is excited.

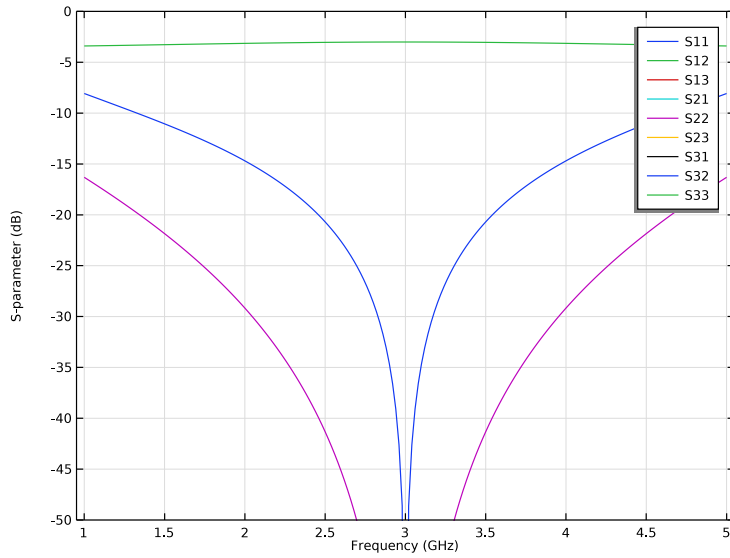


Figure 5: The computed S -parameter plot when port 2 is excited.

References

1. D.M. Pozar, *Microwave Engineering*, John Wiley & Sons, 1998.


2. COMSOL Application Gallery, “Transmission Line Parameter Calculator”, <https://www.comsol.com/model/transmission-line-parameter-calculator-22351>

Application Library path: RF_Module/Couplers_and_Power_Dividers/transmission_line_wpd




Model Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.

MODEL WIZARD

- 1 In the **Model Wizard** window, click  **2D**.
- 2 In the **Select Physics** tree, select **Radio Frequency>Transmission Line (tl)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **General Studies>Frequency Domain**.
- 6 Click  **Done**.

GLOBAL DEFINITIONS

Parameters 1

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:


Name	Expression	Value	Description
f0	3[GHz]	3E9 Hz	Center frequency
l0	250[nH/m]	2.5E-7 H/m	Distributed inductance, 50[ohm] line

Name	Expression	Value	Description
c0	100[pF/m]	1E-10 F/m	Distributed capacitance, 50[ohm] line
z0	sqrt(10/c0)	50 Ω	Characteristic impedance
lda0_t	1/f0/sqrt(10*c0)	0.066667 m	Wavelength, transmission line
l1	10*sqrt(2)	3.5355E-7 H/m	Distributed inductance, 70.71[ohm] line
c1	c0/sqrt(2)	7.0711E-11 F/m	Distributed capacitance, 70.71[ohm] line
z1	sqrt(l1/c1)	70.711 Ω	Characteristic impedance
r0	lda0_t/(4*pi)*360/350	0.0054567 m	Radius, 70.71[ohm] line


GEOMETRY I

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose **mm**.

Line Segment 1 (ls1)


- 1 In the **Geometry** toolbar, click  **More Primitives** and choose **Line Segment**.
- 2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.
- 3 From the **Specify** list, choose **Coordinates**.
- 4 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 5 Locate the **Starting Point** section. In the **x** text field, type -r0-5.
- 6 Locate the **Endpoint** section. In the **x** text field, type -r0.

Circle 1 (c1)

- 1 In the **Geometry** toolbar, click  **Circle**.
- 2 In the **Settings** window for **Circle**, locate the **Object Type** section.
- 3 From the **Type** list, choose **Curve**.
- 4 Locate the **Size and Shape** section. In the **Radius** text field, type r0.
- 5 In the **Sector angle** text field, type 350.

6 Locate the **Rotation Angle** section. In the **Rotation** text field, type 5.

7 Click  **Build Selected**.

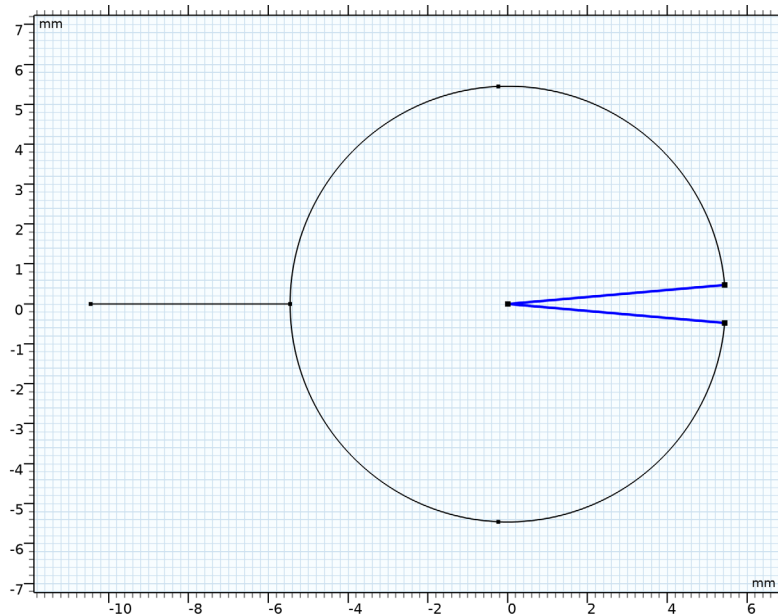
8 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Delete Entities I (delI)

1 In the **Model Builder** window, right-click **Geometry 1** and choose **Delete Entities**.

2 On the object **c1**, select Boundaries 5 and 6 only.

It might be easier to select the boundaries by using the **Selection List** window. To open this window, in the **Home** toolbar click **Windows** and choose **Selection List**. (If you are running the cross-platform desktop, you find **Windows** in the main menu.)



Delete total two line entities.

Circular Arc I (caI)

1 In the **Geometry** toolbar, click  **More Primitives** and choose **Circular Arc**.


2 In the **Settings** window for **Circular Arc**, locate the **Radius** section.

3 In the **Radius** text field, type r0.



4 Locate the **Angles** section. In the **Start angle** text field, type 355.


5 In the **End angle** text field, type 5.

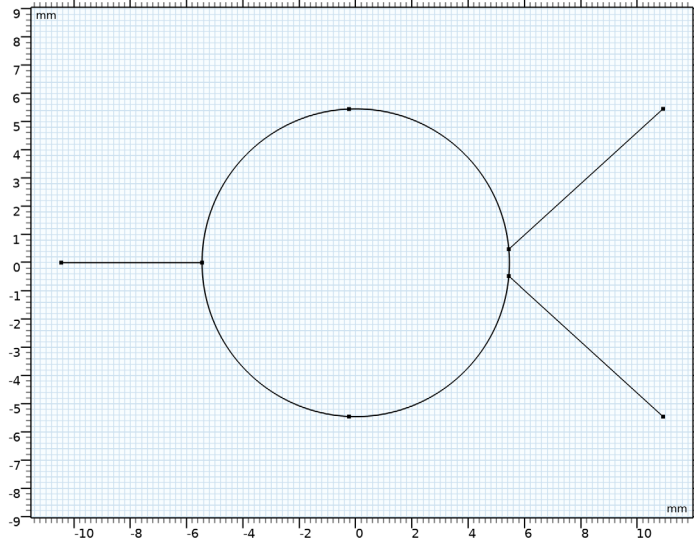
Line Segment 2 (ls2)

- 1 In the **Geometry** toolbar, click  **More Primitives** and choose **Line Segment**.
- 2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.
- 3 From the **Specify** list, choose **Coordinates**.
- 4 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 5 Locate the **Starting Point** section. In the **x** text field, type $r0 \cdot \cos(5[\text{deg}])$.
- 6 In the **y** text field, type $r0 \cdot \sin(5[\text{deg}])$.
- 7 Locate the **Endpoint** section. In the **x** text field, type $r0 \cdot 2$.
- 8 In the **y** text field, type $r0$.

Line Segment 3 (ls3)

- 1 In the **Geometry** toolbar, click  **More Primitives** and choose **Line Segment**.
- 2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.
- 3 From the **Specify** list, choose **Coordinates**.
- 4 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.
- 5 Locate the **Starting Point** section. In the **x** text field, type $r0 \cdot \cos(5[\text{deg}])$.
- 6 In the **y** text field, type $-r0 \cdot \sin(5[\text{deg}])$.
- 7 Locate the **Endpoint** section. In the **x** text field, type $r0 \cdot 2$.
- 8 In the **y** text field, type $-r0$.
- 9 Click  **Build All Objects**.

10 Click the  **Zoom Extents** button in the **Graphics** toolbar.



TRANSMISSION LINE (TL)

Lumped Port 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Transmission Line (tl)** and choose **Lumped Port**.

2 Select Point 1 only.

Lumped Port 2

1 In the **Physics** toolbar, click  **Points** and choose **Lumped Port**.

2 Select Point 8 only.

Lumped Port 3

1 In the **Physics** toolbar, click  **Points** and choose **Lumped Port**.

2 Select Point 7 only.

Transmission Line Equation 1

Set parameter values for $50\ \Omega$ transmission lines. Since this is a default feature, the properties are applied to all line parts.

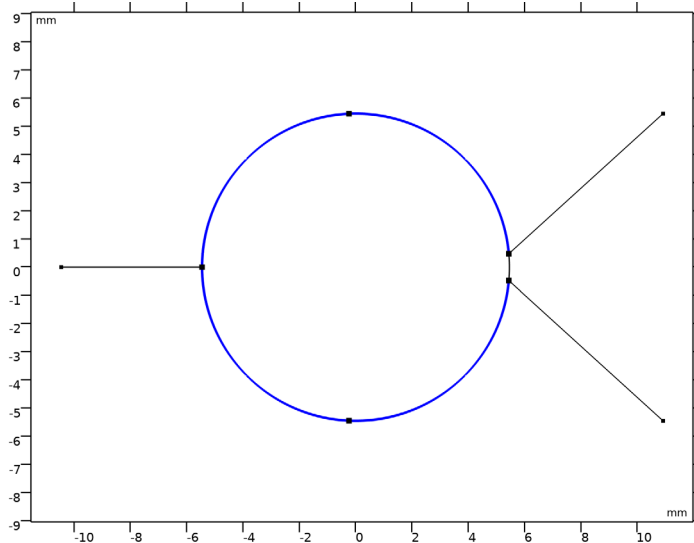
1 In the **Model Builder** window, click **Transmission Line Equation 1**.

2 In the **Settings** window for **Transmission Line Equation**, locate the **Transmission Line Equation** section.

- 3 In the L text field, type 10.
- 4 In the C text field, type c0.

Transmission Line Equation 2

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Transmission Line Equation**.
- 2 Select Boundaries 4–7 only.



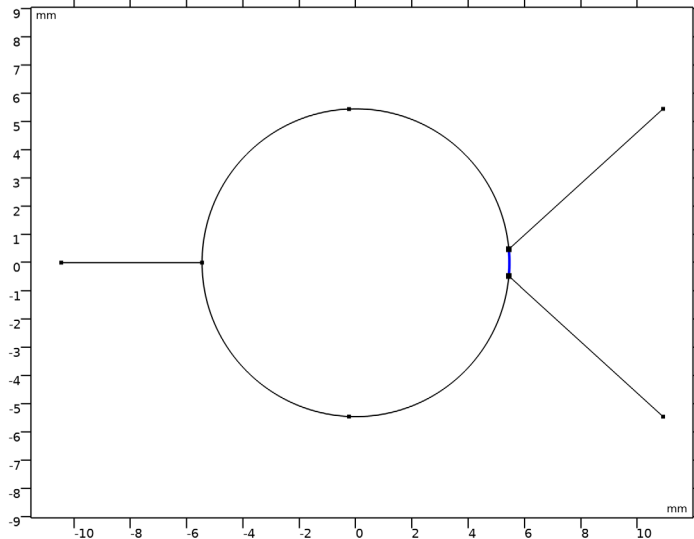
Set parameter values for $70.71 \, \Omega$ transmission lines. This overrides the default properties in the selected boundaries.


- 3 In the **Settings** window for **Transmission Line Equation**, locate the **Transmission Line Equation** section.
- 4 In the L text field, type 11.
- 5 In the C text field, type c1.

Transmission Line Equation 3

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Transmission Line Equation**.

2 Select Boundary 8 only.



3 In the **Geometry** toolbar, click  **Measure**.

Check the Messages window to obtain the measured curvature length and use it to scale the distributed resistance. The transmission line with the adjusted resistance represents a $100\ \Omega$ resistor.

4 In the **Settings** window for **Transmission Line Equation**, locate the **Transmission Line Equation** section.

5 In the R text field, type $100/0.9524e-3$.

6 In the L text field, type 0.

7 In the C text field, type 0.

8 In the **Model Builder** window, click **Transmission Line (tl)**.


9 In the **Settings** window for **Transmission Line**, locate the **Port Sweep Settings** section.

10 Select the **Use manual port sweep** check box.

11 Click **Configure Sweep Settings**. By clicking the **Configure Sweep Settings** button, all necessary port sweep settings such as sweep parameter and parametric study step will be automatically added. It is necessary to run the parametric sweep with port names to get a full S-parameter matrix.

STUDY 1

Step 1: Frequency Domain



- 1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.
- 2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 3 In the **Frequencies** text field, type `range(1[GHz],0.02[GHz],5[GHz])`.
- 4 In the **Home** toolbar, click  **Compute**.

RESULTS

2D Plot Group 1


- 1 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 2 From the **Parameter value (freq (GHz))** list, choose **3**.

Line Graph

- 1 In the **Model Builder** window, expand the **2D Plot Group 1** node, then click **Line Graph**.
- 2 In the **Settings** window for **Line**, locate the **Expression** section.
- 3 In the **Expression** text field, type `20*log10(abs(V))`.
- 4 Locate the **Coloring and Style** section. Click  **Change Color Table**.
- 5 In the **Color Table** dialog box, select **Traffic>Traffic** in the tree.
- 6 Click **OK**.
- 7 In the **2D Plot Group 1** toolbar, click  **Plot**.

The excited electric potential at port 3 is isolated from port 2. Compare with [Figure 2](#).


2D Plot Group 1

- 1 In the **Model Builder** window, click **2D Plot Group 1**.
- 2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 3 From the **Parameter value (PortName)** list, choose **1**.
- 4 In the **2D Plot Group 1** toolbar, click  **Plot**.

When port 1 is excited, the resulting plot shows that the electric potential is distributed equally between port 2 and port 3 ([Figure 3](#)).


S-parameter (tl)

- 1 In the **Model Builder** window, click **S-parameter (tl)**.
- 2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 1/Parametric Solutions 1 (sol2)**.
- 4 From the **Parameter selection (PortName)** list, choose **From list**.

- 5 In the **Parameter values (PortName)** list, select **1**.
- 6 Locate the **Axis** section. Select the **Manual axis limits** check box.
- 7 In the **y minimum** text field, type -50.
- 8 In the **y maximum** text field, type 0.
- 9 In the **S-parameter (tl)** toolbar, click  **Plot**.

The reproduced plot shows the calculated S-parameters when port 1 is excited. See [Figure 4](#).

S-parameter (tl) 1

- 1 Right-click **S-parameter (tl)** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Data** section.
- 3 In the **Parameter values (PortName)** list, select **2**.
- 4 In the **S-parameter (tl) 1** toolbar, click  **Plot**.

The S-parameters plot should look like [Figure 5](#) when port 2 is excited.