Created in COMSOL Multiphysics 6.1

Defining a Mapped Dielectric Distribution of a Material

Introduction

This example demonstrates how to set up a spatially varying dielectric distribution, such as might be engineered with a metamaterial. Here, a convex lens shape is defined via a known deformation of a rectangular domain. The dielectric distribution is defined on the undeformed, original rectangular domain and is mapped onto the deformed shape of the lens. Although the lens shape defined here is convex, the dielectric distribution causes the incident beam to diverge.

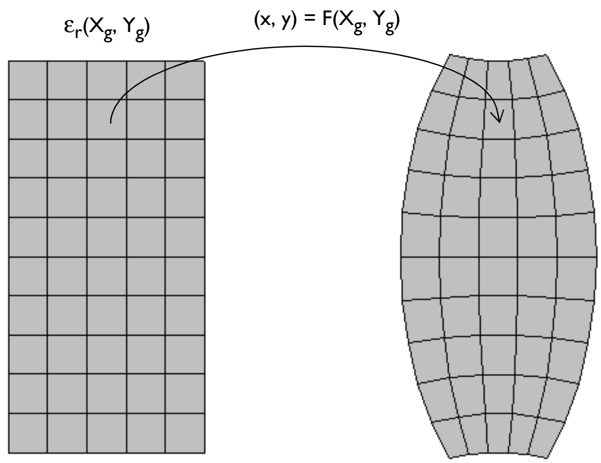


Figure 1: A convex metamaterial lens. Both the shape and the dielectric distribution are defined on a rectangular domain, and mapped into the deformed state.

Model Definition

Consider a 2D model geometry as shown in Figure 2. A square air domain, bounded by a perfectly matched layer (PML) on all sides, encloses a rectangular region in which the metamaterial lens is defined.

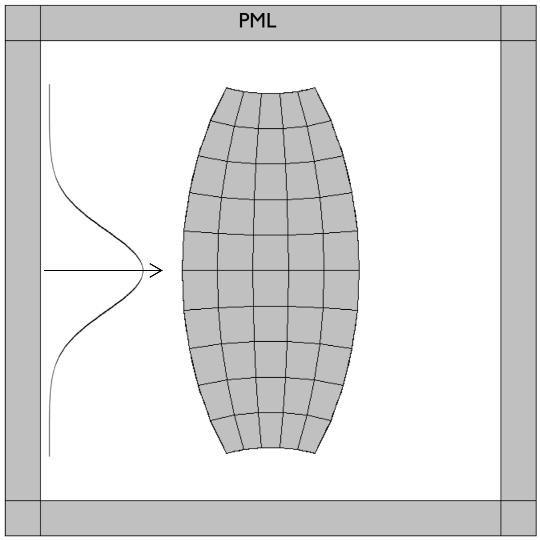


Figure 2: The modeling domain consists of the metamaterial lens in an air domain, and a surrounding PML. A Gaussian beam is incident from the left.

Model a Gaussian beam entering the domain from the left side, via a surface current excitation at an interior boundary. The surface current, J_{s0}, can also be thought of as a displacement current excitation. The waist of the beam is at the boundary, so the excitation at this boundary can be specified as

$$J_{s0} = \exp\left(-\left(\frac{y}{w_0}\right)^2\right)$$

where w_0 is the waist size. The excitation is at the boundary between a domain of free space and the PML, and excites a wave propagating in both directions — into the PML and into the modeling domain. The wave propagating into the PML is completely absorbed, and the wave propagating into the domain is diffracted by the lens.

Both the shape and the dielectric distribution of the metamaterial lens are defined with respect to the original Cartesian coordinate system, as shown in Figure 1. The true shape of the lens is described by the relationship

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} F_x(X_g, Y_g) \\ F_y(X_g, Y_g) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}X_g(2 - Y_g{}^2) \\ Y_g\left(1 + \frac{1}{2}x^2\right) \end{bmatrix}$$

where X_g, Y_g are the Cartesian coordinates of the undeformed frame.

The dielectric distribution is defined on the original Cartesian domain as:

$$\varepsilon_r = \left(1 + \frac{1}{2}Y_g{}^2\right)^2$$

The above expression introduces a variation in the dielectric in the y-coordinate of the undeformed lens. On the deformed lens, the dielectric varies in both directions.

The Deformed Geometry uses the above expressions to define the shape of the lens and maps the Cartesian coordinates of the undeformed frame onto the deformed frame. The dielectric distribution is defined with respect to the undeformed frame, and then mapped onto the deformed shape using the above expressions.

Results and Discussion

The model is solved for the out-of-plane electric field. Figure 3 plots the electric field norm, showing a Gaussian beam with minimal divergence incident upon the lens from the left. The beam is diffracted by the convex lens and spreads out.

Figure 4 displays the dielectric distribution, and shows variation in both directions defined via the mapping described above.

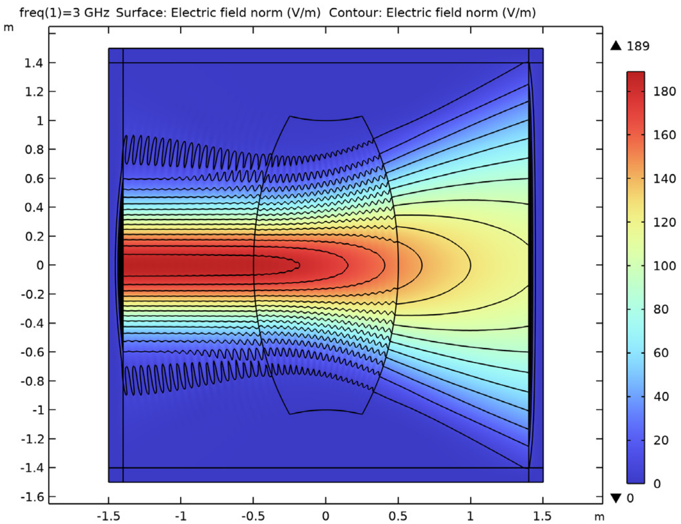


Figure 3: The norm of the electric field shows the Gaussian beam diffracted by the metamaterial lens.

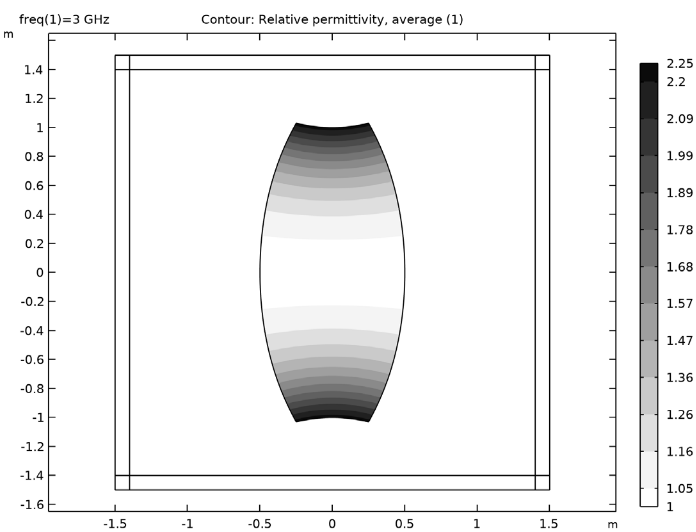


Figure 4: Contour plot of the dielectric distribution.

Application Library path: `RF_Module/Tutorials/mapped_dielectric_distribution`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Radio Frequency>Electromagnetic Waves, Frequency Domain (emw)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Empty Study**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
f0	3[GHz]	3E9 Hz	Operating frequency
lda0	c_const/f0	0.099931 m	Free space wavelength
w0	lda0*4	0.39972 m	Gaussian beam waist size

Here, c_const is a predefined COMSOL constant for the speed of light in vacuum.

GEOMETRY 1

First, create a square for the entire model domain. Add a layer on each side of the square.

Square 1 (sq1)

1 In the **Geometry** toolbar, click **Square**.

2 In the **Settings** window for **Square**, locate the **Size** section.

3 In the **Side length** text field, type `3`.

4 Locate the **Position** section. From the **Base** list, choose **Center**.

5 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
`Layer 1`	`lda0`

6 Select the **Layers to the left** check box.

7 Select the **Layers to the right** check box.

8 Select the **Layers on top** check box.

9 Click **Build Selected**.

Rectangle 1 (r1)

Add a rectangle for the lens.

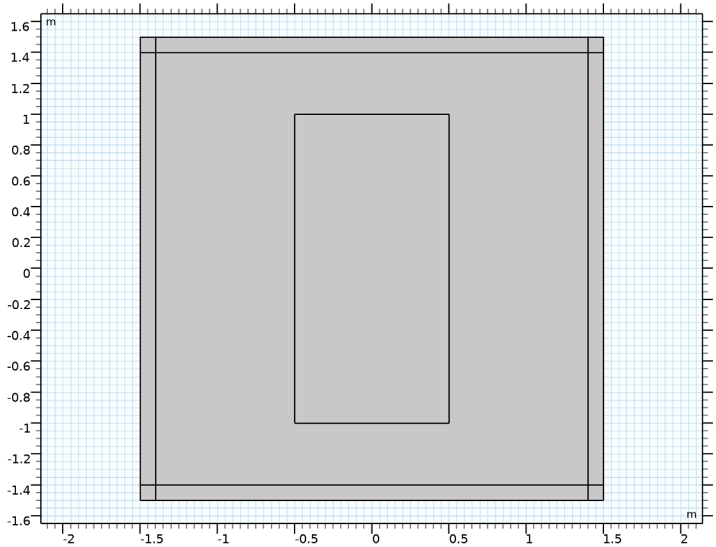
1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Height** text field, type `2`.

4 Locate the **Position** section. From the **Base** list, choose **Center**.

5 Click **Build All Objects**.



DEFINITIONS

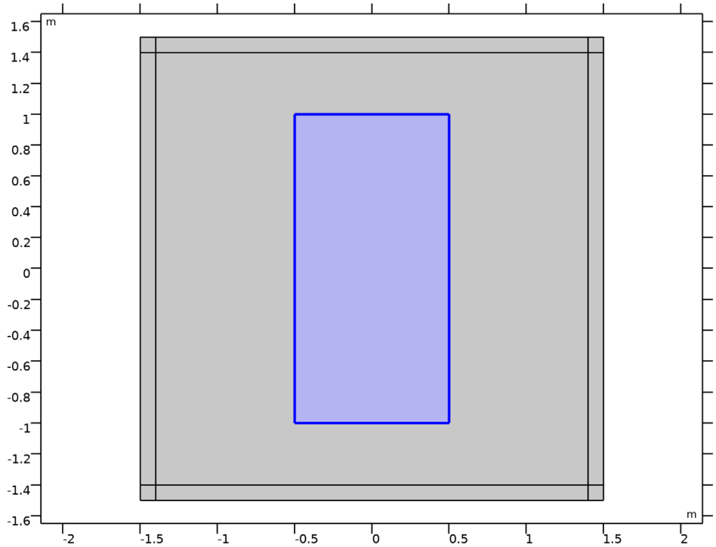
Lens

Add a selection for the lens domain which will be recalled frequently while setting up the model properties.

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Lens` in the **Label** text field.

3 Select Domain 7 only.



Variables 1

Next, add a set of variables for the shape and the dielectric distribution of the lens.

1. In the **Definitions** toolbar, click a= **Local Variables**.
2. In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.
3. From the **Geometric entity level** list, choose **Domain**.
4. From the **Selection** list, choose **Lens**.
5. Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
xp	0.5[m]*Xg[1/m]*(2-(Yg[1/m])^2)	m	Mapping of Xg -> x
yp	Yg*(1+(0.5*(xp[1/m])^2))	m	Mapping of Yg -> y
erp	(1+0.5*(Yg[1/m])^2)^2		Dielectric distribution

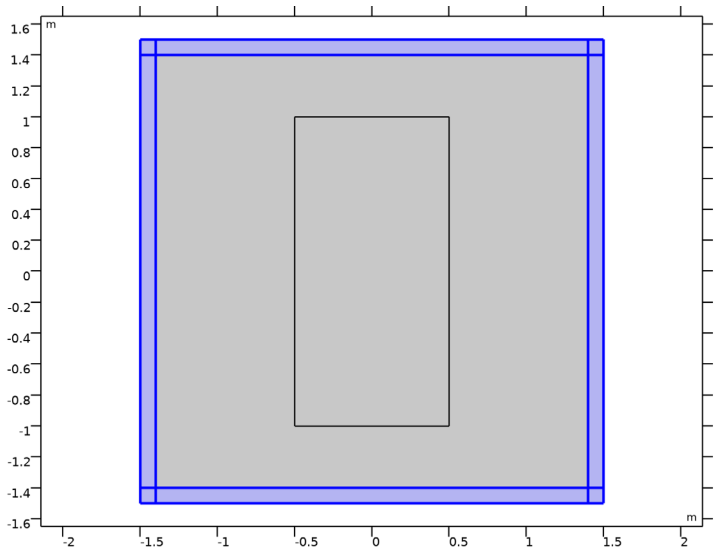
Here, Xg and Yg are predefined Deformed Geometry physics variables representing the Cartesian coordinates of the undeformed frame.

Perfectly Matched Layer 1 (pml1)

Add a perfectly matched layer (PML).

1. In the **Definitions** toolbar, click **Perfectly Matched Layer**.

2 Select Domains 1–4, 6, and 8–10 only.



Set up Deformed geometry. You need to specify Deforming Domain, Prescribed Deformation.

COMPONENT 1 (COMP1)

Deforming Domain 1

1 In the **Definitions** toolbar, click **Deformed Geometry** and choose **Domains> Deforming Domain**.

2 Select Domain 5 only.

3 In the **Settings** window for **Deforming Domain**, locate the **Smoothing** section.

4 From the **Mesh smoothing type** list, choose **Laplace**.

Prescribed Deformation 1

1 In the **Definitions** toolbar, click **Deformed Geometry** and choose **Domains> Prescribed Deformation**.

2 Select Domain 7 only.

3 In the **Settings** window for **Prescribed Deformation**, locate the **Prescribed Deformation** section.

4 Specify the *dx* vector as

xp-Xg	X
yp-Yg	Y

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

In Electromagnetic Waves, Frequency Domain, the dielectric distribution is configured via the user-defined variable `erp` and the Gaussian beam is modeled as entering the domain from the left side, via a surface current excitation.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (emw)**.

2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Components** section.

3 From the **Electric field components solved for** list, choose **Out-of-plane vector** to only perform the calculation for the out-of-plane component. The in-plane components are both zero.

Wave Equation, Electric 2

1 In the **Physics** toolbar, click **Domains** and choose **Wave Equation, Electric**.

2 In the **Settings** window for **Wave Equation, Electric**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Lens**.

4 Locate the **Electric Displacement Field** section. From the ε_r list, choose **User defined**. In the associated text field, type `erp`.

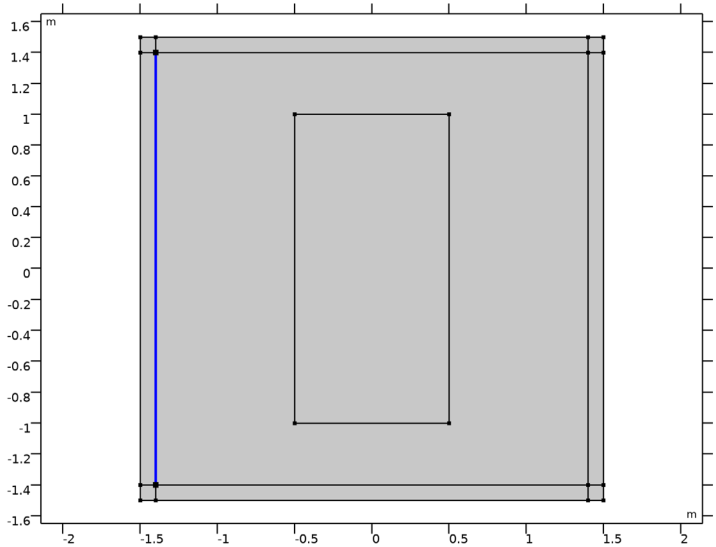
5 Locate the **Magnetic Field** section. From the μ_r list, choose **User defined**. Leave the default value `1`.

6 Locate the **Conduction Current** section. From the σ list, choose **User defined**. Leave the default value `0`.

Surface Current Density 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Surface Current Density**.

2 Select Boundary 10 only.



3 In the **Settings** window for **Surface Current Density**, Specify the $\mathbf{J}_{s0}$ vector as

0	x
0	y
exp(-(y/w0)^2)	z

MATERIALS

Set all domain with vacuum. The lens domain material properties are explicitly configured by Wave Equation, Electric 2 in Electromagnetic Waves, Frequency Domain.

ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

5 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

MESH 1

Free Triangular 1

1 In the **Mesh** toolbar, click **Free Triangular**.

2 In the **Settings** window for **Free Triangular**, locate the **Domain Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domains 5 and 7 only.

Size

1 In the **Model Builder** window, click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 Click the **Custom** button.

4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type `lda0/10`.

5 In the **Minimum element size** text field, type `0.0012`.

Mapped 1

In the **Mesh** toolbar, click **Mapped**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 Select Boundaries 6, 12, 19, and 22 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type `10`.

5 Click **Build All**. You may zoom in a few times to check the quality of the mesh.

The model is analyzed with two study steps. First, make sure that Stationary study step is solved only for Deformed Geometry.

STUDY 1

Stationary

In the **Study** toolbar, click **Study Steps** and choose **Stationary>Stationary**.

Frequency Domain

Add a Frequency Domain study step and set as solved only for Electromagnetic Waves, Frequency Domain.

1 In the **Study** toolbar, click **Study Steps** and choose **Frequency Domain> Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `f0`.

4 Locate the **Physics and Variables Selection** section. In the table, clear the **Solve for** check box for **Deformed geometry (Component 1)**.

5 In the **Study** toolbar, click **Compute**.

RESULTS

Electric Field (emw)

The default plot shows the magnitude of electric fields. Add a contour plot for the magnitude.

Contour 1

1 Right-click **Electric Field (emw)** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Levels** section.

3 In the **Total levels** text field, type 14.

4 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.

5 From the **Color** list, choose **Black**.

6 Clear the **Color legend** check box.

7 In the **Electric Field (emw)** toolbar, click **Plot**. See Figure 3 to compare the reproduced plot.

2D Plot Group 2

Add a filled contour plot describing the dielectric distribution over the lens.

In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

Contour 1

1 Right-click **2D Plot Group 2** and choose **Contour**.

2 In the **Settings** window for **Contour**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Electromagnetic Waves, Frequency Domain>Material properties>emw.epsrAv - Relative permittivity, average**.

3 Locate the **Levels** section. In the **Total levels** text field, type 12.

4 Locate the **Coloring and Style** section. From the **Contour type** list, choose **Filled**.

5 Click **Change Color Table**.

6 In the **Color Table** dialog box, select **Linear>GrayScale** in the tree.

7 Click **OK**.

8 In the **Settings** window for **Contour**, locate the **Coloring and Style** section.

9 From the **Color table transformation** list, choose **Reverse**.

10 In the **2D Plot Group 2** toolbar, click **Plot**.

11 Click the **Zoom Extents** button in the **Graphics** toolbar. The plot for the dielectric distribution is shown in Figure 4.