

# Three-Port Ferrite Circulator

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# Introduction

A microwave circulator is a multiport device that has the property that a wave incident in port 1 is coupled into port 2 only, a wave incident in port 2 is coupled into port 3 only, and so on. A circulator is used to isolate microwave components to couple a transmitter and a receiver to a common antenna, for example. They typically rely on the use of anisotropic materials, most commonly ferrites. In this example, a three-port circulator is constructed from three rectangular waveguide sections joining at 120° where a ferrite post is inserted at the center of the joint. Figure 1 shows the geometry of the circulator.



Figure 1: Geometry of the three-port microwave circulator.

To match the junction, identical dielectric tuning elements are inserted into each branch (not shown above). The ferrite post is magnetized by a static  $H_0$  bias field along the axis. The bias field is usually supplied by external permanent magnets. Here, the focus is on the modeling of the ferrite and how to minimize reflections at the inport by matching the junction by the proper choice of tuning elements. For a general introduction to the modeling of rectangular waveguide structures, see the model H-Bend Waveguide 3D. Matching the circulator junction involves calculating how well a TE<sub>10</sub> wave propagates between ports in the circulator for different materials in the tuning element. This is done by calculating the scattering parameters, or S-parameters, of the structure as a function of the permittivity of the tuning elements for the fundamental TE<sub>10</sub> mode. The S-parameters are a measure of the transmittance and reflectance of the circulator. For a theoretical background on S-parameters, see the section *S-Parameters and Ports* in the *RF Module User's Guide*.

This example only includes the  $TE_{10}$  mode of the waveguide. Thus the model can be made in 2D as the fields of the  $TE_{10}$  mode have no variation in the transverse direction. Figure 2 shows the 2D geometry including the dielectric tuning elements.



Figure 2: 2D geometry with dielectric tuning elements.

# Model Definition

The dependent variable in this physics interface is the z-component of the electric field **E**. It obeys the following relation:

$$\nabla \times (\mu_{\rm r}^{-1} \nabla \times E_z) - (\varepsilon_{\rm r} - \frac{j\sigma}{\omega \varepsilon_0}) k_0^2 E_z = 0$$

where  $\mu_r$  denotes the relative permeability,  $\omega$  the angular frequency,  $\sigma$  the conductivity,  $\varepsilon_0$  the permittivity of vacuum,  $\varepsilon_r$  the relative permittivity, and  $k_0$  is the free space wave number. Losses are neglected so the conductivity is zero everywhere. The magnetic permeability is of key importance in this example as it is the anisotropy of this parameter that is responsible for the nonreciprocal behavior of the circulator. For the theory of the magnetic properties of ferrites, see Ref. 1 and Ref. 2. The model assumes that the static magnetic bias field,  $H_0$ , is much stronger than the alternating magnetic field of the microwaves, so the quoted results are a linearization for a small-signal analysis around this

operating point. Further assume that the applied magnetic bias field is strong enough for the ferrite to be in magnetic saturation. Under these assumptions and neglecting losses, the anisotropic permeability of a ferrite magnetized in the positive *z* direction is given by:

$$[\mu] = \begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$

where

$$\kappa = \mu_0 \left( \frac{\omega \omega_m}{\omega_0^2 - \omega^2} \right)$$
$$\mu = \mu_0 \left( 1 + \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2} \right)$$

and

 $\omega_0 = \mu_0 \gamma H_0$  $\omega_m = \mu_0 \gamma M_s$ 

Here  $\mu_0$  denotes the permeability of free space;  $\omega$  is the angular frequency of the microwave field;  $\omega_0$  is the precession frequency or Larmor frequency of a spinning electron in the applied magnetic bias field,  $H_0$ ;  $\omega_m$  is the electron Larmor frequency at the saturation magnetization of the ferrite,  $M_s$ ; and  $\gamma$  is the gyromagnetic ratio of the electron. For a lossless ferrite, the permeability clearly becomes unbounded at  $\omega = \omega_0$ . In a real ferrite, this resonance becomes finite and is broadened due to losses. For complete expressions including losses, see Ref. 1 and Ref. 2. In this analysis the operating frequency is chosen sufficiently off from the Larmor frequency to avoid the singularity. The material data,  $M_s = 2.39 \cdot 10^5$  A/m and  $\varepsilon_r = 12.9$ , are taken for magnesium ferrite from Ref. 2. The applied bias field is set to  $H_0 = 2.72 \cdot 10^5$  A/m, which is well above saturation. The electron gyromagnetic ratio is set to  $1.759 \cdot 10^{11}$  C/kg. Finally, the model uses an operating frequency of 10 GHz. This is well above the cutoff for the TE<sub>10</sub> mode, which for a waveguide cross section of 2 cm by 1 cm is at about 7.5 GHz. At the ports, matched port boundary conditions make the boundaries transparent to the wave.

The  $S_{11}$  parameter as a function of the relative permittivity of the matching elements, eps\_r, is shown in Figure 3. The  $S_{11}$  parameter corresponds to the reflection coefficient at port 1. Thus matching the junction is equivalent to minimizing the magnitude of  $S_{11}$ .



Figure 3: S<sub>11</sub> parameter as a function of the relative permittivity.

By choosing eps\_r to about 1.29, you obtain a reflection coefficient of about -35 dB, which is a good value for a circulator design. Judging from the absence of standing wave patterns in the magnitude plot of the electric field and by looking at the direction of the

microwave energy flow in the result plot below, it is clear that the circulator behaves as desired.



## References

1. R.E. Collin, *Foundations for Microwave Engineering*, 2nd ed., IEEE Press/Wiley-Interscience, 2000.

2. D.M. Pozar, Microwave Engineering, 3rd ed., John Wiley & Sons, 2004.

Application Library path: RF\_Module/Ferrimagnetic\_Devices/circulator

# Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click 🙆 Model Wizard.

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#### MODEL WIZARD

- I In the Model Wizard window, click **2D**.
- 2 In the Select Physics tree, select Radio Frequency>Electromagnetic Waves, Frequency Domain (emw).
- 3 Click Add.
- 4 Click 🔿 Study.
- 5 In the Select Study tree, select General Studies>Frequency Domain.
- 6 Click 🗹 Done.

## GLOBAL DEFINITIONS

#### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
eps_r	1	I	Relative permittivity

## GEOMETRY I

Import I (imp1)

- I In the **Home** toolbar, click 🔚 Import.
- 2 In the Settings window for Import, locate the Import section.
- 3 Click **Browse**.
- **4** Browse to the model's Application Libraries folder and double-click the file circulator.mphbin.
- 5 Click া Import.
- 6 Click the **Click the Zoom Extents** button in the **Graphics** toolbar.

#### ADD MATERIAL

- I In the Home toolbar, click 🙀 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

## MATERIALS

Air (mat1)

Select Domains 1, 3, 6, and 7 only.

Define the dielectric matching elements with a permittivity eps\_r. You will later set up the solver to sweep this parameter.

## Dielectric

- I In the Model Builder window, right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Dielectric in the Label text field.
- **3** Select Domains 2, 4, and 5 only.
- 4 Locate the Material Contents section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	eps_r	I	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

Ferrite

- I Right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Ferrite in the Label text field.
- **3** Select Domain 8 only.

**4** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	12.9	I	Basic
Relative permeability	{mur11, mur21, mur31, mur12, mur22, mur32, mur13, mur23, mur33}	{mur,-i*kr,0,i* kr,mur,0,0,0,1}	I	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

- 5 In the Model Builder window, expand the Component 1 (comp1)>Materials>Ferrite (mat3) node, then click Basic (def).
- 6 In the Settings window for Basic, locate the Model Inputs section.
- 7 Click + Select Quantity.
- 8 In the Physical Quantity dialog box, type frequency in the text field.
- 9 Click 🔫 Filter.
- IO In the tree, select General>Frequency (Hz).
- II Click OK.

To define mur and kr in terms of the frequency, you need to enter a number of local parameters.

12 In the Settings window for Basic, locate the Local Properties section.

**I3** In the **Local properties** table, enter the following settings:

Name	Expression	Unit	Description
gamma	1.759e11[C/kg]	C/kg	
omega	2*pi*freq	Hz	

Name	Expression	Unit	Description
H0	omega/(gamma*mu0_const+1e4[m/C])	A/m	
wO	mu0_const*gamma*H0	l/s	
Ms	0.3[Wb/m^2]/mu0_const	A/m	
wm	mu0_const*gamma*Ms	l/s	
mur	1+w0*wm/(w0^2-omega^2)		
kr	omega*wm/(w0^2-omega^2)		

## ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

With the Electromagnetic Waves interface selected and the materials defined, the physics you need to specify is only the ports.

- I In the Model Builder window, under Component I (compl) click Electromagnetic Waves, Frequency Domain (emw).
- **2** In the Settings window for Electromagnetic Waves, Frequency Domain, locate the Components section.
- 3 From the Electric field components solved for list, choose Out-of-plane vector.

Only the *z*-component of electric field (transverse electric mode) is effective in the simulation domain. By choosing the **Out-of-plane vector**, the computation can be more efficient.

Port I

- I In the Physics toolbar, click Boundaries and choose Port.
- **2** Select Boundary 1 only.
- 3 In the Settings window for Port, locate the Port Properties section.
- **4** From the **Type of port** list, choose **Rectangular**.

For the first port, wave excitation is **on** by default.

## Port 2

- I In the Physics toolbar, click Boundaries and choose Port.
- 2 Select Boundary 20 only.
- 3 In the Settings window for Port, locate the Port Properties section.
- 4 From the Type of port list, choose Rectangular.

## Port 3

- I In the Physics toolbar, click Boundaries and choose Port.
- **2** Select Boundary 21 only.

- 3 In the Settings window for Port, locate the Port Properties section.
- 4 From the Type of port list, choose Rectangular.

## MESH I

- I In the Model Builder window, under Component I (compl) click Mesh I.
- 2 In the Settings window for Mesh, locate the Physics-Controlled Mesh section.
- 3 In the table, clear the Use check box for Electromagnetic Waves, Frequency Domain (emw).
- 4 From the Element size list, choose Finer.
- 5 Click 📗 Build All.

#### STUDY I

Parametric Sweep

- I In the Study toolbar, click **Parametric Sweep**.
- 2 In the Settings window for Parametric Sweep, locate the Study Settings section.
- 3 Click + Add.
- 4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
eps_r (Relative permittivity)	range(1,0.01,1.5)	

#### Step 1: Frequency Domain

- I In the Model Builder window, click Step I: Frequency Domain.
- 2 In the Settings window for Frequency Domain, locate the Study Settings section.
- **3** In the **Frequencies** text field, type 10[GHz].
- **4** In the **Study** toolbar, click **= Compute**.

#### RESULTS

#### Electric Field (emw)

The default plot shows the norm of the electric field for  $eps_r = 1.5$ . The plot is dominated by the strong field in the ferrite post. Adjust the range to get a better overview of the fields throughout the circulator, and add arrows representing the power flow.

#### Surface

I In the Model Builder window, expand the Electric Field (emw) node, then click Surface.

2 In the Settings window for Surface, click to expand the Range section.

- **3** Select the Manual color range check box.
- 4 In the Maximum text field, type 350.
- 5 In the Electric Field (emw) toolbar, click 🗿 Plot.

#### Arrow Surface 1

- I In the Model Builder window, right-click Electric Field (emw) and choose Arrow Surface.
- 2 In the Settings window for Arrow Surface, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)> Electromagnetic Waves, Frequency Domain>Energy and power>emw.Poavx,emw.Poavy Power flow, time average.
- **3** Locate the **Arrow Positioning** section. Find the **Y grid points** subsection. In the **Points** text field, type **25**.
- 4 Locate the Coloring and Style section. From the Color list, choose Black.
- 5 In the Electric Field (emw) toolbar, click 💽 Plot.

The presence of standing waves in the input arm is clearly visible for this value (1.5) of the relative permittivity in the matching elements. To study how the reflections depend on the value of this parameter, plot the reflection coefficient S<sub>11</sub> as a function of eps\_r.

Global I

- I In the Model Builder window, expand the Results>S-parameter (emw) node, then click Global I.
- In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
  Electromagnetic Waves, Frequency Domain>Ports>S-parameter, dB>emw.SlldB Sll.
- 3 In the S-parameter (emw) toolbar, click 💽 Plot.

The plot shows that  $eps_r = 1.29$  gives the minimum reflection. You can study the field distribution for this solution by selecting it as follows:

Electric Field (emw)

- I In the Model Builder window, under Results click Electric Field (emw).
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Parameter value (eps\_r) list, choose 1.29.
- **4** In the **Electric Field (emw)** toolbar, click **O Plot**.