Created in COMSOL Multiphysics 6.1

Mooney-Rivlin Curve Fit

Introduction

This tutorial model demonstrates how to use the Optimization Module to estimate unknown function parameters based on measured data. The two-parameter Mooney-Rivlin solid material model is used as an example, but the procedure is generally applicable when you need to fit a parameterized analytic function to measured data.

Note: This application is also used in *Introduction to the Optimization Module*.

Model Definition

The two-parameter incompressible Mooney–Rivlin material model describes the local behavior of rubber-like materials. The model assumes that the local strain energy density in an incompressible solid is a simple function of local strain invariants.

In a standard tensile test, a rotationally symmetric test specimen is pulled in such a way that it extends in one direction and contracts symmetrically in the other two. For this case of uniaxial extension, the relationship between applied force, F, and resulting extension, ΔL, of a true Mooney–Rivlin material is

$$\frac{F}{A_0} = 2\left(C_{10} + C_{01}\frac{L_0}{L_0 + \Delta L}\right)\left(\frac{L_0 + \Delta L}{L_0} - \left(\frac{L_0}{L_0 + \Delta L}\right)^2\right) \qquad (1)$$

where A_0 is the original cross-section area of the test specimen and L_0 is its reference length. The constants C_{10} and C_{01} are material parameters that must be determined by fitting Equation 1 to the experimental data from the tensile test.

In practice, tensile test data is delivered in a form which is independent of the geometry of the test specimen used. There are multiple possible formats. The one used here contains corresponding measured values of engineering stress, P_i, representing force per unit reference area

$$P = \frac{F}{A_0}$$

and stretch, λ_i, representing relative elongation

$$\lambda = \frac{L_0 + \Delta L}{L_0}$$

The expected relationship between these variables for a Mooney–Rivlin material is

$$P(\lambda) = 2\left(C_{10} + \frac{C_{01}}{\lambda}\right)\left(\lambda - \frac{1}{\lambda^2}\right)$$

Given N pairs of measurements (λ_i, P_i), $i = 1, \ldots, N$, the values of C_{10} and C_{01} that best fit the measured data are considered to be those which minimize the total squared error

$$e = \sum_{i=1}^{N} (P(\lambda_i) - P_i)^2$$

The curve-fitting problem is therefore identical to an optimization problem.

Results

Figure 1 shows measured stress and stress computed using the material properties that have been fitted to the measured data.

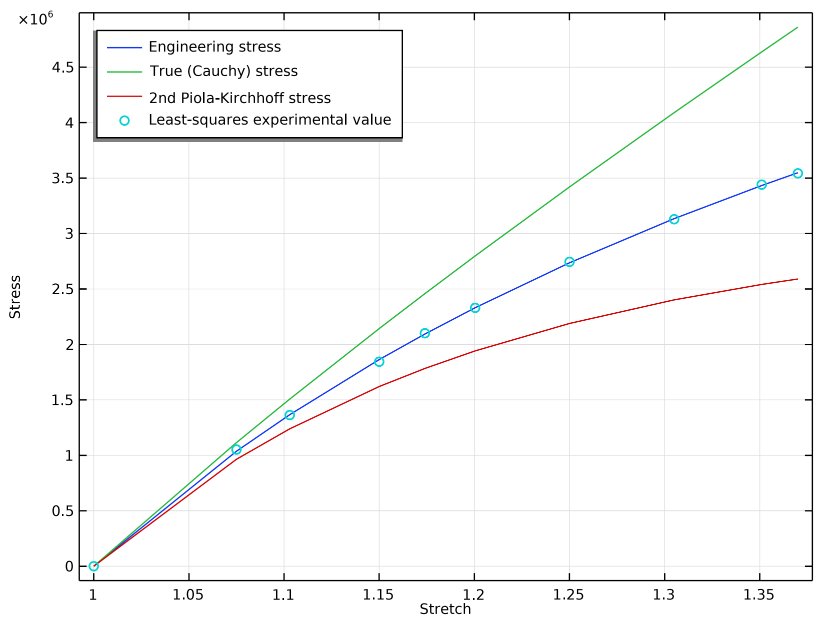

Figure 1: Measured (turquoise circles) and computed (blue line) engineering stresses. In addition, two other stress measures are plotted for comparison.

It should be noted that for large stretches, the values of the different stress measures will differ significantly. For a uniaxial stress state and an incompressible material like in this case, the true stress (Cauchy stress; stress per current area) is related to the engineering stress by

$$\sigma(\lambda) = \lambda P(\lambda) \tag{2}$$

The 2nd Piola-Kirchhoff stress is defined as

$$S(\lambda) = \frac{P(\lambda)}{\lambda} \tag{3}$$

It is thus necessary to pay attention to the stress measures actually used when a interpreting results from an actual simulation in which a material model for large stretches is used.

Application Library path: `Optimization_Module/Parameter_Estimation/curve_fit_mooney_rivlin`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Blank Model**.

ADD COMPONENT

In the **Home** toolbar, click **Add Component** and choose **0D**.

GLOBAL DEFINITIONS

Add the stretch parameter `lambda` which has been varied in the measured data. Also add the unknown material parameters as global model parameters.

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	**Expression**	**Value**	**Description**
`lambda`	`1`	1	`Stretch`
`C10`	`1[MPa]`	1E6 Pa	`Mooney-Rivlin parameter`
`C01`	`1[MPa]`	1E6 Pa	`Mooney-Rivlin parameter`

Variables 1

1 In the **Home** toolbar, click **Variables** and choose **Global Variables**.

Set up the assumed relationship between stretch and engineering stress as a variable expression in terms of `lambda` and the yet unknown `C10` and `C01`.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	**Expression**	**Unit**	**Description**
`P`	`2*(C10+C01/lambda)*(lambda-1/lambda^2)`	Pa	`Engineering stress`

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Stationary**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

Use a **Parameter Estimation** study step to set up control variables and select an optimization solver. The **Levenberg-Marquardt** solver is particularly efficient for least-squares problems such as this one.

STUDY 1

Parameter Estimation

1 In the **Study** toolbar, click **Optimization** and choose **Parameter Estimation**.

2 In the **Settings** window for **Parameter Estimation**, locate the **Experimental Data** section.

3 Click the **Browse** button. From the menu, choose **Browse**.

4 Browse to the model's Application Libraries folder and double-click the file `curve_fit_mooney_rivlin.csv`.

5 Locate the **Column Settings** section. In the table, click to select the cell at row number 1 and column number 2.

6 In the table, enter the following settings:

Columns	Type	Settings
Column 1	Parameter	Name=lambda
Column 2	Value	Model expression=1, Name=col2, Weight=1

7 From the **Name** list, choose **lambda (Stretch)**.

8 In the **Unit** text field, type `1`.

9 In the table, click to select the cell at row number 2 and column number 3.

10 In the **Model expression** text field, type `P`.

11 In the **Name** text field, type `engStress`.

12 In the **Unit** text field, type `Pa`.

13 Locate the **Parameters** section. Click **Add** twice

14 In the table, enter the following settings:

Parameter name	Initial value	Scale	Lower bound	Upper bound
C01 (Mooney-Rivlin parameter)	`1[MPa]`	`1[MPa]`		
C10 (Mooney-Rivlin parameter)	`1[MPa]`	`1[MPa]`		

15 Locate the **Parameter Estimation Method** section. From the **Method** list, choose **Levenberg-Marquardt**.

16 Find the **Solver settings** subsection. From the **Least-squares time/parameter method** list, choose **From least-squares objective**.

17 In the **Study** toolbar, click **Compute**.

RESULTS

Add a plot of the least-squares fitted stress-strain curve together with the measured data.

Parameter Estimation

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Parameter Estimation` in the **Label** text field.

Global 1

1 Right-click **Parameter Estimation** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Global definitions>Variables>P - Engineering stress - Pa**.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
`P*lambda`	`Pa`	`True (Cauchy) stress`
`P/lambda`	`Pa`	`2nd Piola-Kirchhoff stress`

Global 2

1 In the **Model Builder** window, right-click **Parameter Estimation** and choose **Global**.

2 In the **Settings** window for **Global**, click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Solver>Parameter estimation> opt.glsobj.engStress.data - Least-squares experimental value - Pa**.

3 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

4 Find the **Line markers** subsection. From the **Marker** list, choose **Circle**.

5 In the **Parameter Estimation** toolbar, click **Plot**.

Parameter Estimation

Finish the plot by adjusting the title, axis labels, and legend positioning.

1 In the **Model Builder** window, click **Parameter Estimation**.

2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section.

5 Select the **x-axis label** check box. In the associated text field, type `Stretch`.

6 Select the **y-axis label** check box. In the associated text field, type `Stress`.

7 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

Evaluation Group 1

In the **Results** toolbar, click **Evaluation Group**.

Parameter Estimation

Click the **Zoom Extents** button in the **Graphics** toolbar.

Global Evaluation 1

In the **Model Builder** window, right-click **Evaluation Group 1** and choose **Global Evaluation**.

Evaluation Group 1

1 In the **Settings** window for **Evaluation Group**, locate the **Data** section.

2 From the **Parameter selection (lambda)** list, choose **Last**.

Global Evaluation 1

Use the predefined **Objective value** node to evaluate the estimated values of material parameters `C01` and `C10`.

1 In the **Model Builder** window, click **Global Evaluation 1**.

2 In the **Settings** window for **Global Evaluation**, click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Solver>Control parameters> C10 - Mooney-Rivlin parameter - Pa**.

3 Click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Solver>Control parameters>C01 - Mooney-Rivlin parameter - Pa**.

4 In the **Evaluation Group 1** toolbar, click **= Evaluate**.