

Diffraction Patterns

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Introduction

This example simulates a double-slit interference experiment with water waves or sound. The model mimics the incoming plane-wave excitation with two thin waveguides leading to slits in a screen and computes the diffraction pattern on the other side of the screen.

Model Definition



Figure 1: Model geometry with inflow and outflow boundaries indicated. On all other boundaries, a zero flux condition applies.

Theory predicts amplitude minima along rays where the difference in travel distance is an odd multiple of half the wavelength, and maxima at even multiples. For $n = 0, \pm 1, \pm 2, ...$

$$\begin{cases} \min, \sin \theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{D} \\ \max, \sin \theta = n\frac{\lambda}{D} \end{cases}$$

In this example, the distance *D* between the slits is 2λ . Maxima should then be found at $\theta = 0^{\circ}$ and 30° , while minima should appear at $\theta = 14.48^{\circ}$ and 48.59° .

Equation

For time-harmonic propagation, the wave equation turns into the Helmholtz equation:

$$-\nabla \cdot (\nabla u) - k^2 u = 0, \qquad k = \frac{2\pi}{\lambda}$$

Boundary Conditions

On the inflow and outflow boundaries (see Figure 1), absorbing boundary conditions apply. Let us briefly show how such conditions in their simplest form can be derived. First, assume the solution at the boundaries to be the sum of an incident plane wave, u_{in} , propagating in an arbitrary direction and a scattered wave, u_{sc} , propagating in the normal direction:

$$u = u_{\rm in} + u_{\rm sc} = u_0 e^{-i\mathbf{k}\cdot\mathbf{x}} + u_s e^{-ik\mathbf{n}\cdot\mathbf{x}}$$
(1)

Here **n** is the outward boundary normal vector and $k \equiv |\mathbf{k}|$. At the boundary of the modeling domain, Γ , we then have

$$\nabla u|_{\Gamma} = -i\mathbf{k}u_0 e^{-i\mathbf{k}\cdot\mathbf{x}} - ik\mathbf{n}u_s e^{-ik\mathbf{n}\cdot\mathbf{x}} = -ik\mathbf{n}u|_{\Gamma} - iu_0(\mathbf{k} - k\mathbf{n})e^{-i\mathbf{k}\cdot\mathbf{x}}$$
(2)

where Equation 1 was used in the second step. It follows that

$$\mathbf{n} \cdot \nabla u|_{\Gamma} = -iku|_{\Gamma} + iu_0(k - \mathbf{n} \cdot \mathbf{k})e^{-i\mathbf{k} \cdot \mathbf{x}}$$
(3)

There is no incident wave on the outflow boundary, which means that the second term on the right-hand side of Equation 3 vanishes. For the inflow boundary, make the further approximation that the incident wave propagates in the inward normal direction, so that $\mathbf{k} = -k\mathbf{n}$. We then arrive at the following boundary conditions:

$$\mathbf{n} \cdot \nabla u|_{\Gamma} = -iku|_{\Gamma} + 2iku_0 e^{ik\mathbf{n} \cdot \mathbf{x}}, \text{ inflow}$$

$$\mathbf{n} \cdot \nabla u|_{\Gamma} = -iku|_{\Gamma}, \qquad \text{outflow}$$

$$(4)$$

In this model, these conditions are readily imposed using the Coefficient Form PDE interface's Flux/Source condition. The default condition, Zero Flux, applies on the remaining boundaries.

Results and Discussion

The plot in Figure 2 shows the diffraction pattern clearly. The effect of discretization is that the numerical wavelength differs from λ , which results in a shift of the angles. You can correct for this effect by adjusting the value of *k* in the Helmholtz equation to the element



size. These practices are important for modeling the interference effects of monochromatic waves.

Figure 2: Diffraction pattern in the simulated double-slit experiment.

Application Library path: COMSOL_Multiphysics/Equation_Based/

diffraction_patterns

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click 🙆 Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **Q** 2D.
- 2 In the Select Physics tree, select Mathematics>PDE Interfaces>Coefficient Form PDE (c).
- 3 Click Add.

4 Click 🔿 Study.

5 In the Select Study tree, select General Studies>Stationary.

6 Click 🗹 Done.

GLOBAL DEFINITIONS

Parameters 1

I In the Model Builder window, under Global Definitions click Parameters I.

2 In the Settings window for Parameters, locate the Parameters section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
1	0.1[m]	0.1 m	Wavelength
k	2*pi[rad]/l	62.832 rad/m	Wave number
u0	1	I	Incident wave amplitude at inflow boundaries

GEOMETRY I

Circle 1 (c1)

- I In the **Geometry** toolbar, click \bigcirc **Circle**.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- **3** In the **Sector angle** text field, type 180.
- 4 Locate the **Position** section. In the **x** text field, type 0.5.
- 5 Locate the Rotation Angle section. In the Rotation text field, type -90.
- 6 Click 틤 Build Selected.

Rectangle 1 (r1)

- I In the **Geometry** toolbar, click **Rectangle**.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 0.5.
- 4 In the **Height** text field, type 0.03.
- 5 Locate the **Position** section. In the **y** text field, type -0.015-0.1.
- 6 Click 틤 Build Selected.

Copy I (copyI)

I In the Geometry toolbar, click 💭 Transforms and choose Copy.

- 2 Select the object rl only.
- 3 In the Settings window for Copy, locate the Displacement section.
- 4 In the **y** text field, type 0.2.
- 5 Click 틤 Build Selected.

Union I (uni I)

- I In the Geometry toolbar, click 🔲 Booleans and Partitions and choose Union.
- 2 Click in the Graphics window and then press Ctrl+A to select all objects.
- 3 In the Settings window for Union, locate the Union section.
- 4 Clear the Keep interior boundaries check box.
- 5 In the Geometry toolbar, click 🟢 Build All.
- 6 Click the **Zoom Extents** button in the **Graphics** toolbar.

The model geometry is now complete. Next, turn to the physics settings.

COEFFICIENT FORM PDE (C)

Coefficient Form PDE 1

- I In the Model Builder window, under Component I (compl)>Coefficient Form PDE (c) click Coefficient Form PDE 1.
- 2 In the Settings window for Coefficient Form PDE, locate the Absorption Coefficient section.
- **3** In the *a* text field, type $-k^2$.
- **4** Locate the **Source Term** section. In the *f* text field, type **0**.

Proceed to apply the boundary conditions Equation 4 at the inflow and outflow boundaries.

Flux/Source 1

- I In the Physics toolbar, click Boundaries and choose Flux/Source.
- **2** Select Boundaries 1 and 4 only.
- 3 In the Settings window for Flux/Source, locate the Boundary Flux/Source section.
- **4** In the *g* text field, type -i*k*u+2*u0*i*k*exp(-i*k*x).

Note that the phase factor in this expression evaluates to 1 because x = 0. It has been included nevertheless for completeness.

Flux/Source 2

- I In the Physics toolbar, click Boundaries and choose Flux/Source.
- 2 Select Boundaries 11 and 12 only.

3 In the Settings window for Flux/Source, locate the Boundary Flux/Source section.

4 In the *g* text field, type -i*k*u.

MESH I

Create a mesh with a maximum element size determined by the wavelength. As a rule of thumb, you need 5 elements per wavelength for quadratic elements (the default for the PDE interface) to fully resolve the wave.

Free Triangular 1

In the **Mesh** toolbar, click **Free Triangular**.

Size

- I In the Model Builder window, click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- **3** Click the **Custom** button.
- 4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type 1/5.



5 Click 📗 Build All.

STUDY I

In the **Home** toolbar, click **= Compute**.

RESULTS

2D Plot Group 1

To reproduce the plot shown in Figure 2, just change the color table.

Surface 1

- I In the Model Builder window, expand the 2D Plot Group I node, then click Surface I.
- 2 In the Settings window for Surface, locate the Coloring and Style section.
- 3 Click Change Color Table.
- 4 In the Color Table dialog box, select Wave>WaveLight in the tree.
- 5 Click OK.
- 6 In the 2D Plot Group I toolbar, click 💽 Plot.