



Optimization of a Tesla Microvalve

Introduction

A Tesla valve inhibits backward flow on a fixed geometry by utilizing friction forces instead of moving parts, [Ref. 1](#). This means fluid can flow freely in one direction but not in the reverse direction. Typically the Reynolds number of the flow in microfluidics is up to 100.

Model Definition

The model solves two instances of the Navier-Stokes equations, one for the forward flow and one for the reverse. The Reynolds number is 100 in this example. A measure of the effectiveness of the design is the ratio of the pressure drop between the inlet and outlet for the forward and reverse flow. The pressure drop is defined as:

$$\Delta p_{\text{forward}} = \int_{\text{inlet}} p dS - \int_{\text{outlet}} p dS$$

For the reverse flow the same expression is used, except the inlet and outlet correspond to different boundaries in the model. The ratio of the pressure drop between the reverse and forward flow is then:

$$D_i = \frac{\Delta p_{\text{backward}}}{\Delta p_{\text{forward}}}$$

Unfortunately this expression does not form a well posed objective function, so an alternative expression is required for the optimization problem. According to [Ref. 2](#), the energy dissipation is a well posed objective function for topological optimization:

$$\text{obj} = \int_{\Omega} (\boldsymbol{\tau} : \mathbf{S} + \alpha(\mathbf{u} \cdot \mathbf{u})) dV$$

where $\boldsymbol{\tau}$ is the viscous stress and $\mathbf{S}$ is the strain rate tensor:

$$\mathbf{S} = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$$

The fluid flow is described by the Navier-Stokes equations:

$$\begin{aligned} \rho(\mathbf{u} \cdot \nabla) \mathbf{u} &= -\nabla p + \nabla \cdot \eta(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \alpha(\theta) \mathbf{u} \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}$$

where the coefficient $\alpha(\theta)$ depends on the distribution of material which impedes the flow within the device. In this example, $\alpha(\theta)$ is given by:

$$\alpha(\theta_p) = \alpha_0 \theta_p, \quad \theta_p = \frac{\alpha_0(q + \theta)}{q + \theta}$$

$$\theta = \frac{(\tanh(\beta(\theta_f - \theta_\beta)) + \tanh(\beta\theta_\beta))}{(\tanh(\beta(1 - \theta_\beta)) + \tanh(\beta\theta_\beta))}$$

$$\theta_f = R_{\min}^2 \nabla^2 \theta_f + \theta_c$$

where θ_c and θ_β , θ and θ_p are the control- and filtered material volume factor. To avoid the effect of grayscale the filtered field is projected to construct the material volume factor, θ , which is related to the damping term using a convex function, see [Ref. 3](#). This method is used to avoid grayscale and spurious holes as seen in [Figure 1](#).

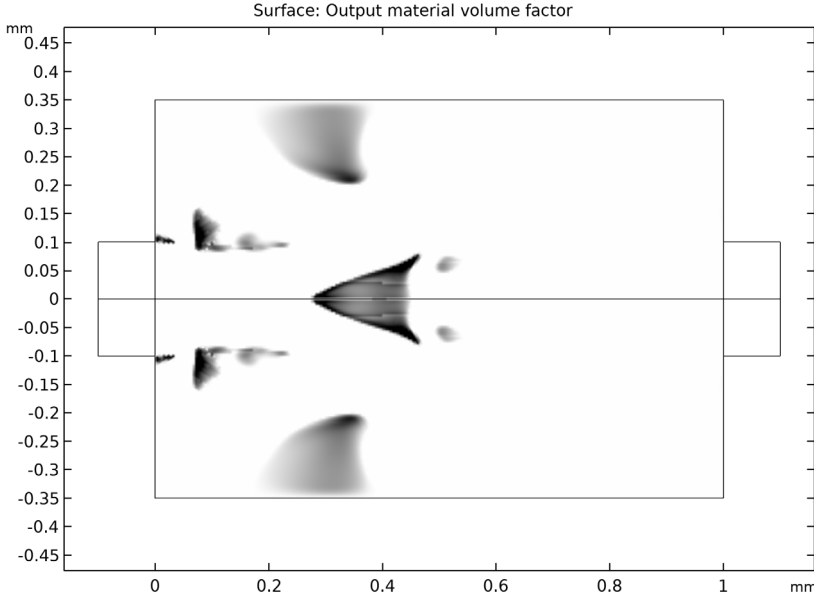


Figure 1: Optimizing without a filter gives a design with interpretation issues.

Results and Discussion

As expected, the forward and reverse flow are identical but in opposite directions for the initial topology; see [Figure 2](#) and [Figure 3](#). The forward flow in the optimized design after 200 iterations can be seen in [Figure 4](#). Material has been added close to the outlet in a triangular shape that makes the forward flow bend smoothly around it. This smooth diversion of the flow from the point of impingement results in an overall low pressure drop

between the inlet and the outlet. The reverse flow, shown in [Figure 5](#), is far more interesting. The triangular shaped obstacle has a flat edge normal to the incident fluid, which means the velocity is redirected upward and downward toward the exterior walls. The redirected flow is then directed toward additional obstacles that further impede the flow path.

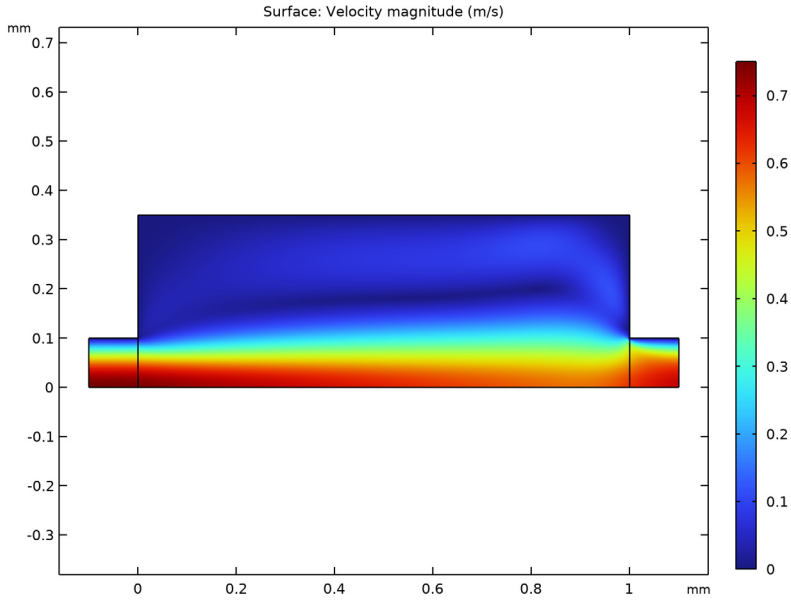


Figure 2: Forward flow velocity field, initial geometry.

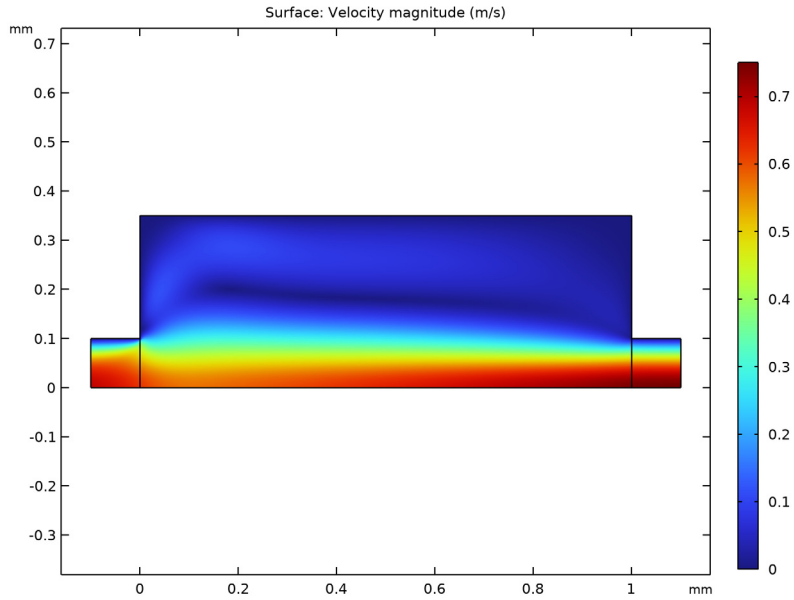


Figure 3: Reverse flow velocity field, initial geometry.

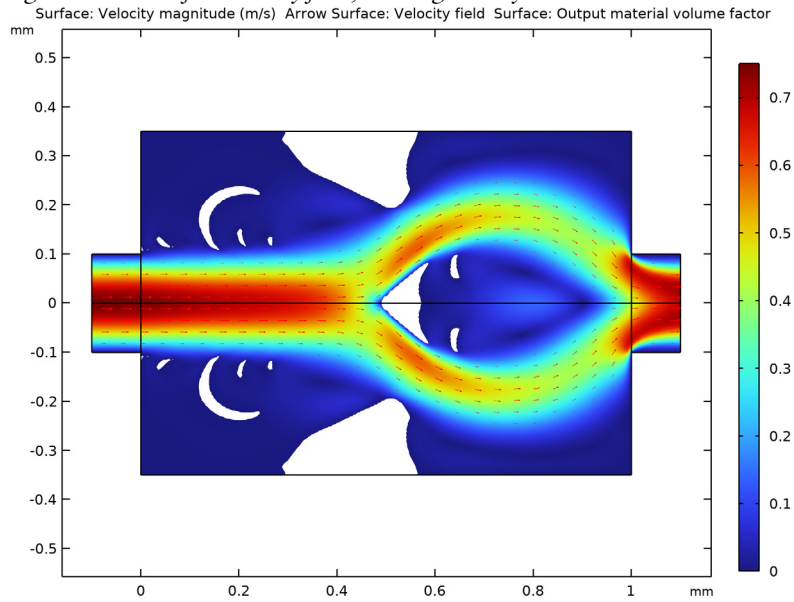


Figure 4: Forward flow, optimized topology.

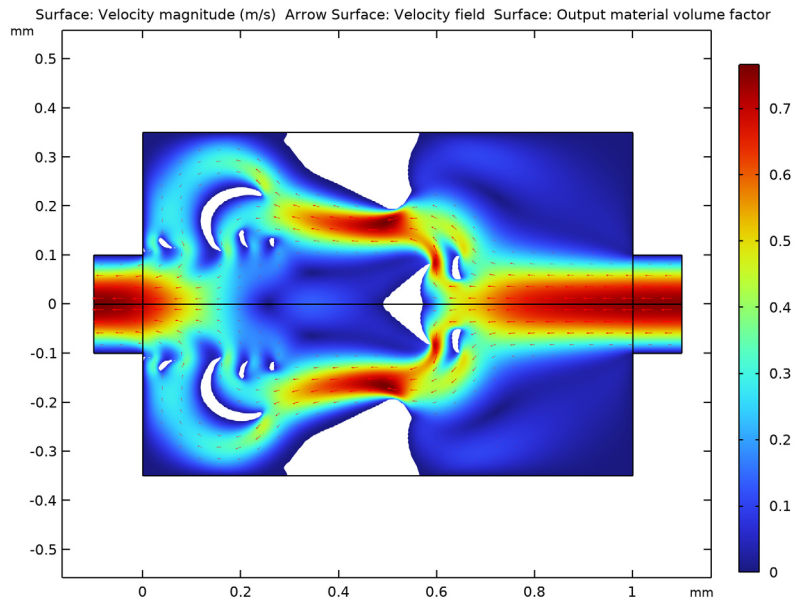


Figure 5: Reverse flow, optimized topology.

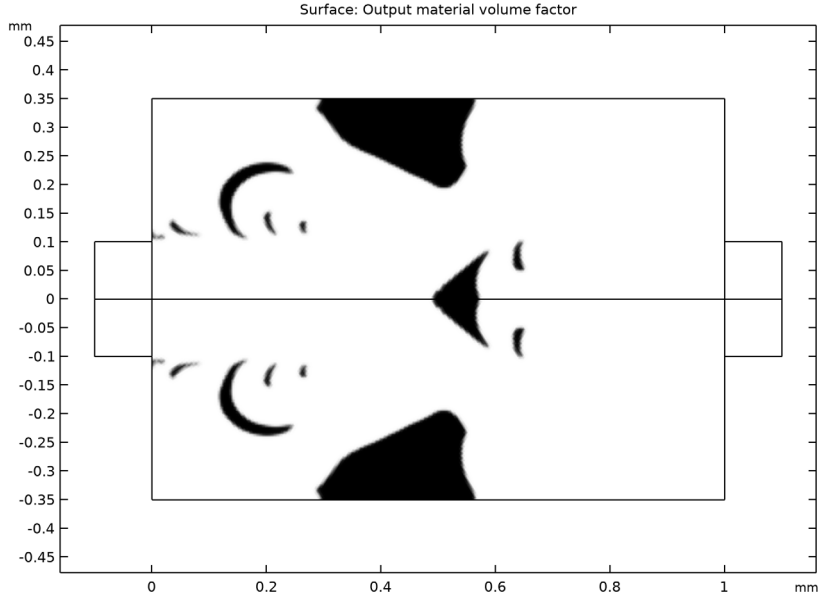


Figure 6: Control variable after optimization.

Finally, the optimized results is transferred to a new component for verification. This results in a larger diodicity.

References

1. S. Lin, "Topology Optimization of Micro Tesla Valve in low and moderate Reynolds number," Chinese Academy of Sciences, China, September 27, 2011.
2. L. Højgaard Olesen, F. Okkels, and H. Bruus, "A high-level programming-language implementation of topology optimization applied to steady-state Navier–Stokes flow," *Int. J. Numer. Methods Eng.*, vol. 65, pp. 975–1001, 2006.
3. T. Borrvall and J. Petersson, "Topology optimization of fluids in Stokes flow," *Int. J. Numer. Meth. Fluid*, vol. 41, pp. 77–107, 2003.

Notes About the COMSOL Implementation

The model is set up using two Laminar Flow interfaces, one for the forward flow and one for the reverse. The original MMA optimization solver (which is not globally convergent) is used.

Application Library path: Microfluidics_Module/Fluid_Flow/
tesla_microvalve_optimization

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.

MODEL WIZARD

- 1 In the **Model Wizard** window, click  **2D**.
- 2 In the **Select Physics** tree, select **Fluid Flow>Single-Phase Flow>Laminar Flow (spf)**.
Add two **Laminar Flow** interfaces for the forward and backward flow, respectively.
- 3 Click **Add**.
- 4 In the **Select Physics** tree, select **Fluid Flow>Single-Phase Flow>Laminar Flow (spf)**.
- 5 Click **Add**.
- 6 Click  **Study**.
- 7 In the **Select Study** tree, select **General Studies>Stationary**.
- 8 Click  **Done**.

GEOMETRY I

The geometry is on the order of millimeters, so change the geometry unit.

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose **mm**.

Add some parameters for computing the average inlet velocity for the flow, based on a chosen Reynolds number, in this case 100.

GLOBAL DEFINITIONS

Parameters 1

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
Re	100	100	Reynolds number
D	0.2[mm]	2E-4 m	Characteristic dimension
L	5*D	0.001 m	Length of channel
H	1.75*D	3.5E-4 m	Width of channel
mu0	1E-3[Pa*s]	0.001 Pa*s	Dynamic viscosity
rho0	1E3[kg/m^3]	1000 kg/m ³	Density
Uin	Re*mu0/(rho0*D)	0.5 m/s	Average inlet velocity
meshsz	0.005*L	5E-6 m	Mesh size

GEOMETRY 1

Rectangle 1 (r1)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type L.
- 4 In the **Height** text field, type H.

Rectangle 2 (r2)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type D/2.
- 4 In the **Height** text field, type D/2.
- 5 Locate the **Position** section. In the **x** text field, type -D/2.
- 6 Click  **Build Selected**.

Rectangle 3 (r3)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

- 3 In the **Width** text field, type $D/2$.
- 4 In the **Height** text field, type $D/2$.
- 5 Locate the **Position** section. In the **x** text field, type L .

Symmetry

- 1 In the **Geometry** toolbar, click  **Selections** and choose **Box Selection**.
- 2 In the **Settings** window for **Box Selection**, type **Symmetry** in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Box Limits** section. In the **y maximum** text field, type $1e3*eps$.
- 5 Locate the **Output Entities** section. From the **Include entity if** list, choose **Entity inside box**.

Left

- 1 In the **Geometry** toolbar, click  **Selections** and choose **Box Selection**.
- 2 In the **Settings** window for **Box Selection**, type **Left** in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Box Limits** section. In the **x maximum** text field, type $-D/2+1e3*eps$.
- 5 Locate the **Output Entities** section. From the **Include entity if** list, choose **Entity inside box**.

Right

- 1 In the **Geometry** toolbar, click  **Selections** and choose **Box Selection**.
- 2 In the **Settings** window for **Box Selection**, type **Right** in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Box Limits** section. In the **x minimum** text field, type $L+D/2-1e3*eps$.
- 5 Locate the **Output Entities** section. From the **Include entity if** list, choose **Entity inside box**.
- 6 Click  **Build All Objects**.
- 7 Click the  **Zoom Extents** button in the **Graphics** toolbar.

GLOBAL DEFINITIONS

Add a blank material for ρ_0 and μ_0 .

Material 1 (mat1)

- 1 In the **Model Builder** window, under **Global Definitions** right-click **Materials** and choose **Blank Material**.

- 2 In the **Materials** toolbar, click  **User-Defined Property Group**.
- 3 In the **Settings** window for **Basic**, locate the **Output Properties** section.
- 4 Click  **Select Quantity**.
- 5 In the **Physical Quantity** dialog box, type density in the text field.
- 6 Click  **Filter**.
- 7 In the tree, select **General>Density (kg/m³)**.
- 8 Click **OK**.
- 9 In the **Settings** window for **Basic**, locate the **Output Properties** section.
- 10 In the table, enter the following settings:

Property	Variable	Expression	Unit	Size
Density	rho	rho0	kg/m ³	1x1

- 11 Click  **Select Quantity**.
- 12 In the **Physical Quantity** dialog box, type viscosity in the text field.
- 13 Click  **Filter**.
- 14 In the tree, select **Transport>Dynamic viscosity (Pa*s)**.
- 15 Click **OK**.
- 16 In the **Settings** window for **Basic**, locate the **Output Properties** section.
- 17 In the table, enter the following settings:

Property	Variable	Expression	Unit	Size
Dynamic viscosity	mu	mu0	Pa*s	1x1

MATERIALS

Material Link 1 (matlnk1)

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **More Materials>Material Link**.

LAMINAR FLOW (SPF)

Inlet 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Laminar Flow (spf)** and choose **Inlet**.
- 2 In the **Settings** window for **Inlet**, locate the **Boundary Selection** section.

- 3 From the **Selection** list, choose **Left**.
- 4 Locate the **Boundary Condition** section. From the list, choose **Fully developed flow**.
- 5 Locate the **Fully Developed Flow** section. In the U_{av} text field, type U_{in} .

Outlet 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Outlet**.
- 2 In the **Settings** window for **Outlet**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Right**.

Wall 2

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Wall**.
- 2 In the **Settings** window for **Wall**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Symmetry**.
- 4 Locate the **Boundary Condition** section. From the **Wall condition** list, choose **Slip**.

LAMINAR FLOW 2 (SPF2)

In the **Model Builder** window, under **Component 1 (comp1)** click **Laminar Flow 2 (spf2)**.

Inlet 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Inlet**.
- 2 In the **Settings** window for **Inlet**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Right**.
- 4 Locate the **Boundary Condition** section. From the list, choose **Fully developed flow**.
- 5 Locate the **Fully Developed Flow** section. In the U_{av} text field, type U_{in} .

Outlet 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Outlet**.
- 2 In the **Settings** window for **Outlet**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Left**.

Wall 2

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Wall**.
- 2 In the **Settings** window for **Wall**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Symmetry**.
- 4 Locate the **Boundary Condition** section. From the **Wall condition** list, choose **Slip**.

MESH I

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
- 2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.
- 3 From the **Element size** list, choose **Extremely fine**.

Free Triangular 1

In the **Mesh** toolbar, click  **Free Triangular**.

Size

- 1 In the **Model Builder** window, click **Size**.
- 2 In the **Settings** window for **Size**, click to expand the **Element Size Parameters** section.
- 3 Locate the **Element Size** section. Click the **Custom** button.
- 4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type meshsz.
- 5 Click  **Build All**.

STUDY I

In the **Home** toolbar, click  **Compute**.

RESULTS

Mirror 2D 1

- 1 In the **Results** toolbar, click  **More Datasets** and choose **Mirror 2D**.
- 2 In the **Settings** window for **Mirror 2D**, locate the **Axis Data** section.
- 3 In row **Point 2**, set **x** to 1 and **y** to 0.

Velocity (spf)

- 1 In the **Model Builder** window, under **Results** click **Velocity (spf)**.
- 2 In the **Velocity (spf)** toolbar, click  **Plot**.
- 3 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Velocity (spf2)

- 1 In the **Model Builder** window, click **Velocity (spf2)**.
- 2 In the **Velocity (spf2)** toolbar, click  **Plot**.

Now define two average operators on the inlet and outlet of the modeling domain. These will be used to compute the pressure ratio between the inlet and outlet for the forward and reverse flow.

DEFINITIONS

Average 1 (aveop1)

- 1 In the **Definitions** toolbar, click  **Nonlocal Couplings** and choose **Average**.
- 2 In the **Settings** window for **Average**, locate the **Source Selection** section.
- 3 From the **Geometric entity level** list, choose **Boundary**.
- 4 From the **Selection** list, choose **Left**.

Average 2 (aveop2)

- 1 In the **Definitions** toolbar, click  **Nonlocal Couplings** and choose **Average**.
- 2 In the **Settings** window for **Average**, locate the **Source Selection** section.
- 3 From the **Geometric entity level** list, choose **Boundary**.
- 4 From the **Selection** list, choose **Right**.

Integration 1 (intop1)

- 1 In the **Definitions** toolbar, click  **Nonlocal Couplings** and choose **Integration**.
- 2 Select Domain 2 only.

Define a variable corresponding to the ratio of the pressure difference between the forward and backward flow.

Variables 1

- 1 In the **Definitions** toolbar, click  **Local Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Variables** section.
- 3 In the table, enter the following settings:

Name	Expression	Unit	Description
dP_forward	aveop1(p) - aveop2(p)	Pa	Pressure difference, forward direction
dP_backward	aveop2(p2) - aveop1(p2)	Pa	Pressure difference, backward direction
Di	dP_backward/dP_forward		Ratio of pressure differences

In order to evaluate the ratio of the pressure differences, the model needs updating. Note that the model does not need to be solved again.

STUDY 1

In the **Study** toolbar, click  **Update Solution**.

RESULTS

Diodicity

- 1 In the **Results** toolbar, click  **Evaluation Group**.
- 2 In the **Settings** window for **Evaluation Group**, type Diodicity in the **Label** text field.

Global Evaluation 1

- 1 Right-click **Diodicity** and choose **Global Evaluation**.
- 2 In the **Settings** window for **Global Evaluation**, click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Definitions>Variables>Di - Ratio of pressure differences**.
- 3 In the **Diodicity** toolbar, click  **Evaluate**.

TABLE

- 1 Go to the **Table** window.

Observe that the ratio of the pressure differences is very close to 1. This concludes solving of the forward problem; now set up the optimization problem.

Add a density topology feature, which can be used to distinguish between free flow and solid regions. This variable will be coupled back to the **Laminar Flow** interfaces later. The filter radius should not be smaller than the mesh element size. The default value will work, but a fixed value has to be chosen to make the result of the optimization mesh independent.

COMPONENT 1 (COMP1)

Density Model 1 (dtopo1)

- 1 In the **Definitions** toolbar, click  **Optimization** and choose **Topology Optimization>Density Model**.
Only the center part of the channel geometry is needed in the optimization, so you only have to define the feature there.
- 2 Select Domain 2 only.
Use a filter to prevent a checkerboard instability that may otherwise occur for the control variable theta.
- 3 In the **Settings** window for **Density Model**, locate the **Interpolation** section.
- 4 From the **Interpolation type** list, choose **Darcy**.
- 5 Locate the **Projection** section. From the **Projection type** list, choose **Hyperbolic tangent projection**.

- 6 Locate the **Interpolation** section. In the q_{Darcy} text field, type 1.
- 7 Locate the **Control Variable Initial Value** section. In the θ_0 text field, type 1.
- 8 Locate the **Control Variable Discretization** section. From the **Element order** list, choose **Constant**.

Now the design variable used in the optimization is defined. The initial value 1 corresponds to a channel free from porous material.

Now define the friction force to be used in the **Laminar Flow** interfaces in the form of viscous and friction dissipation that can be integrated over the domain to obtain a suitable objective function.

DEFINITIONS

Variables 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)>Definitions** click **Variables 1**.
- 2 In the **Settings** window for **Variables**, locate the **Variables** section.
- 3 In the table, enter the following settings:

Name	Expression	Unit	Description
phi_forward	$\text{spf.Qvd} + \alpha * (u^2 + v^2)$	W/m ³	Dissipation density, forward flow
phi_backward	$\text{spf2.Qvd} + \alpha * (u^2 + v^2)$	W/m ³	Dissipation density, backward flow
phi_total	$\text{phi_backward} + \text{phi_forward}$	W/m ³	Total dissipation
E_forward	$\text{intop1}(\text{phi_forward})$	W/m	Energy dissipation, forward flow
E_backward	$\text{intop1}(\text{phi_backward})$	W/m	Energy dissipation, backward flow
obj	$\text{E_backward} / \text{E_forward}$		Objective function
alpha	$16 * \mu_0 * \text{dtopo1.theta_p} / \text{meshsz}^2$	Pa·s/m ²	Friction force

LAMINAR FLOW (SPF)

Add the friction force to the **Laminar Flow** interfaces so that the fluid flows around regions where θ is 0 and through regions where θ is 1.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Laminar Flow (spf)**.

Volume Force 1

1 In the **Physics** toolbar, click  **Domains** and choose **Volume Force**.

2 Select Domain 2 only.

3 In the **Settings** window for **Volume Force**, locate the **Volume Force** section.

4 Specify the **F** vector as

-alpha*u	x
-alpha*v	y

LAMINAR FLOW 2 (SPF2)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Laminar Flow 2 (spf2)**.

2 In the **Physics** toolbar, click  **Domains** and choose **Volume Force**.

Volume Force 1

1 Select Domain 2 only.

2 In the **Settings** window for **Volume Force**, locate the **Volume Force** section.

3 Specify the **F** vector as

-alpha*u2	x
-alpha*v2	y

OPTIMIZATION

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, type Optimization in the **Label** text field.

Topology Optimization

1 In the **Study** toolbar, click  **Optimization** and choose **Topology Optimization**.

2 In the **Settings** window for **Topology Optimization**, locate the **Optimization Solver** section.

3 In the **Maximum number of iterations** text field, type 200.

4 Select the **Move limits** check box. In the associated text field, type 0.2.

5 Click **Add Expression** in the upper-right corner of the **Objective Function** section. From the menu, choose **Component 1 (comp1)>Definitions>Variables>comp1.obj - Objective function**.

6 Locate the **Objective Function** section. From the **Type** list, choose **Maximization**.

7 In the **Study** toolbar, click  **Get Initial Value**.

Solver Configurations

In the **Model Builder** window, expand the **Optimization>Solver Configurations** node.

Solution 1 (sol1)

The old version of **MMA** (1987) is less prone to premature convergence.

- 1 In the **Model Builder** window, expand the **Optimization>Solver Configurations>Solution 1 (sol1)** node, then click **Optimization Solver 1**.
- 2 In the **Settings** window for **Optimization Solver**, locate the **Optimization Solver** section.
- 3 Clear the **Globally Convergent MMA** check box.

RESULTS

Mirror 2D 1

Use the mirrored dataset to plot the value of theta during the optimization.

Topology

- 1 In the **Home** toolbar, click  **Add Plot Group** and choose **2D Plot Group**.
- 2 In the **Settings** window for **2D Plot Group**, type Topology in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Mirror 2D 1**.

Surface 1

- 1 Right-click **Topology** and choose **Surface**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type `dtopo1.theta`.
- 4 Locate the **Coloring and Style** section. Click  **Change Color Table**.
- 5 In the **Color Table** dialog box, select **Linear>GrayScale** in the tree.
- 6 Click **OK**.
- 7 In the **Settings** window for **Surface**, locate the **Coloring and Style** section.
- 8 Clear the **Color legend** check box.
- 9 Click to expand the **Range** section. Select the **Manual color range** check box.
- 10 In the **Maximum** text field, type 1.
- 11 In the **Topology** toolbar, click  **Plot**.

OPTIMIZATION

Topology Optimization

- 1 In the **Model Builder** window, under **Optimization** click **Topology Optimization**.

- 2 In the **Settings** window for **Topology Optimization**, locate the **Output While Solving** section.
- 3 Select the **Plot** check box.
- 4 From the **Plot group** list, choose **Topology**.
- 5 In the **Home** toolbar, click  **Compute**.

RESULTS

Forward Flow

The forward and backward flow patterns computed using the optimization solver can now be visualized.

- 1 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 2 From the **Dataset** list, choose **Mirror 2D 1**.
- 3 In the **Velocity (spf)** toolbar, click  **Plot**.
- 4 In the **Label** text field, type Forward Flow.

Arrow Surface 1

- 1 Right-click **Forward Flow** and choose **Arrow Surface**.
- 2 In the **Settings** window for **Arrow Surface**, locate the **Arrow Positioning** section.
- 3 Find the **x grid points** subsection. In the **Points** text field, type 30.
- 4 Find the **y grid points** subsection. In the **Points** text field, type 30.
- 5 In the **Forward Flow** toolbar, click  **Plot**.

Surface 2

- 1 Right-click **Forward Flow** and choose **Surface**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type `dtopo1.theta`.
- 4 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.
- 5 From the **Color** list, choose **White**.
- 6 Locate the **Range** section. Select the **Manual data range** check box.
- 7 In the **Maximum** text field, type 0.5.
- 8 In the **Forward Flow** toolbar, click  **Plot**.
- 9 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Now follow a similar procedure for the backward flow.

Arrow Surface 1, Surface 2

- 1 In the **Model Builder** window, under **Results>Forward Flow**, Ctrl-click to select **Arrow Surface 1** and **Surface 2**.
- 2 Right-click and choose **Copy**.

Backward Flow

- 1 In the **Model Builder** window, under **Results** click **Velocity (spf2)**.
- 2 In the **Settings** window for **2D Plot Group**, type Backward Flow in the **Label** text field.

Arrow Surface 1

- 1 Right-click **Backward Flow** and choose **Paste Multiple Items**.
- 2 In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Laminar Flow 2>Velocity and pressure>u2,v2 - Velocity field**.
- 3 In the **Backward Flow** toolbar, click  **Plot**.

Backward Flow

- 1 In the **Model Builder** window, click **Backward Flow**.
- 2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Mirror 2D 1**.
- 4 In the **Backward Flow** toolbar, click  **Plot**.
- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar.
- 6 In the **Backward Flow** toolbar, click  **Plot**.

Topology

- 1 In the **Model Builder** window, click **Topology**.
- 2 In the **Topology** toolbar, click  **Plot**.
- 3 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Backward Flow, Forward Flow, Pressure (spf), Pressure (spf2), Topology

- 1 In the **Model Builder** window, under **Results**, Ctrl-click to select **Forward Flow**, **Pressure (spf)**, **Backward Flow**, **Pressure (spf2)**, and **Topology**.
- 2 Right-click and choose **Group**.

Optimized Design

In the **Settings** window for **Group**, type Optimized Design in the **Label** text field.

Next, evaluate the damping with respect to the relative amount of dissipation in the solid region to see if the optimization result can be trusted.

Dissipation in Solid Regions

- 1 In the **Results** toolbar, click  **Evaluation Group**.
- 2 In the **Settings** window for **Evaluation Group**, type Dissipation in Solid Regions in the **Label** text field.

Global Evaluation 1

- 1 Right-click **Dissipation in Solid Regions** and choose **Global Evaluation**.
- 2 In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.
- 3 In the table, enter the following settings:

Expression	Unit	Description
$\text{intop1}(\text{phi_total} * (1 - \text{dtopo1}.\text{theta})) / \text{intop1}(\text{phi_total})$	1	Relative dissipation in solid

- 4 In the **Dissipation in Solid Regions** toolbar, click  **Evaluate**.
There is significant power loss in the solid regions, which is unphysical. Therefore it is a good idea to set up a verification analysis in a new component.

The **Density Model** generates a **Filter Dataset** when the default plots are created. However, since **Density Model** was added later, a **Filter Dataset** must be added manually.

Filter 1

- 1 In the **Results** toolbar, click  **More Datasets** and choose **Filter**.
- 2 In the **Settings** window for **Filter**, locate the **Expression** section.
- 3 In the **Expression** text field, type `dtopo1.theta`.
- 4 Locate the **Filter** section. In the **Lower bound** text field, type `0.5`.
- 5 Locate the **Evaluation** section. Clear the **Use derivatives** check box.
Create a new component from the filter dataset.
- 6 Right-click **Filter 1** and choose **Create Mesh in New Component**.

MESH 2

Import 1

- 1 In the **Settings** window for **Import**, locate the **Import** section.
- 2 Click **Import**.

Adapt 1

- 1 In the **Mesh** toolbar, click  **Modify** and choose **Adapt**.

- 2 In the **Settings** window for **Adapt**, locate the **Adaptation** section.
- 3 From the **Solution** list, choose **None**.
- 4 From the **Type of expression** list, choose **Absolute size**.
- 5 In the **Size expression** text field, type `meshsz`.
- 6 In the **Maximum number of refinements** text field, type 0.
- 7 In the **Maximum coarsening factor** text field, type `Inf`.
- 8 Click  **Build Selected**.

MATERIALS

Material Link 2 (matlnk2)

In the **Model Builder** window, under **Component 2 (comp2)** right-click **Materials** and choose **More Materials>Material Link**.

LAMINAR FLOW (SPF), LAMINAR FLOW 2 (SPF2)

- 1 In the **Model Builder** window, under **Component 1 (comp1)**, Ctrl-click to select **Laminar Flow (spf)** and **Laminar Flow 2 (spf2)**.

Copy and paste the physics from the first component and fix the selections.

- 2 Right-click and choose **Copy**.

LAMINAR FLOW (SPF3)

In the **Model Builder** window, right-click **Component 2 (comp2)** and choose **Paste Multiple Items**.

LAMINAR FLOW (SPF3), LAMINAR FLOW 2 (SPF4)

- 1 In the **Model Builder** window, under **Component 2 (comp2)**, Ctrl-click to select **Laminar Flow (spf3)** and **Laminar Flow 2 (spf4)**.

- 2 In the **Messages from Paste** dialog box, click **OK**.

Inlet 1

- 1 In the **Model Builder** window, expand the **Component 2 (comp2)>Laminar Flow (spf3)** node, then click **Inlet 1**.

- 2 In the **Settings** window for **Inlet**, locate the **Boundary Selection** section.

- 3 From the **Selection** list, choose **Left**.

Outlet 1

- 1 In the **Model Builder** window, click **Outlet 1**.

- 2 In the **Settings** window for **Outlet**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Right**.

Wall 2

- 1 In the **Model Builder** window, click **Wall 2**.
- 2 In the **Settings** window for **Wall**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Symmetry**.

Volume Force 1

In the **Model Builder** window, under **Component 2 (comp2)>Laminar Flow (spf3)** right-click **Volume Force 1** and choose **Delete**.

LAMINAR FLOW 2 (SPF4)

Inlet 1

- 1 In the **Model Builder** window, expand the **Component 2 (comp2)>Laminar Flow 2 (spf4)** node, then click **Inlet 1**.
- 2 In the **Settings** window for **Inlet**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Right**.

Outlet 1

- 1 In the **Model Builder** window, click **Outlet 1**.
- 2 In the **Settings** window for **Outlet**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Left**.

Wall 2

- 1 In the **Model Builder** window, click **Wall 2**.
- 2 In the **Settings** window for **Wall**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Symmetry**.

Volume Force 1

In the **Model Builder** window, under **Component 2 (comp2)>Laminar Flow 2 (spf4)** right-click **Volume Force 1** and choose **Delete**.

DEFINITIONS (COMP2)

Set up a new average operator so that a new diodicity variable can be defined for the new component.

Average 3 (aveop3)

- 1 In the **Definitions** toolbar, click  **Nonlocal Couplings** and choose **Average**.

- 2 In the **Settings** window for **Average**, locate the **Source Selection** section.
- 3 From the **Geometric entity level** list, choose **Boundary**.
- 4 From the **Selection** list, choose **Left**.

Average 4 (aveop4)

- 1 Right-click **Average 3 (aveop3)** and choose **Duplicate**.
- 2 In the **Settings** window for **Average**, locate the **Source Selection** section.
- 3 From the **Selection** list, choose **Right**.

Variables 2

- 1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Variables** section.
- 3 In the table, enter the following settings:

Name	Expression	Unit	Description
Di	$\frac{(\text{aveop4}(p2) - \text{aveop3}(p2))}{(\text{aveop3}(p) - \text{aveop4}(p))}$		Ratio of pressure differences

ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.
- 4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check boxes for **Laminar Flow (spf)** and **Laminar Flow 2 (spf2)**.
- 5 Click **Add Study** in the window toolbar.
- 6 In the **Model Builder** window, click the root node.
- 7 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

OPTIMIZATION

Step 1: Stationary

- 1 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.
- 2 In the table, clear the **Solve for** check boxes for **Laminar Flow (spf3)** and **Laminar Flow 2 (spf4)**.

STUDY 2

Step 1: Stationary

- 1 In the **Model Builder** window, under **Study 2** click **Step 1: Stationary**.
- 2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.
- 3 In the table, clear the **Solve for** check box for **Topology Optimization (Component 1)**.
- 4 In the **Model Builder** window, click **Study 2**.
- 5 In the **Settings** window for **Study**, type **Verification** in the **Label** text field.
- 6 In the **Home** toolbar, click  **Compute**.

RESULTS

Pressure (spf3), Pressure (spf4), Velocity (spf3), Velocity (spf4)

- 1 In the **Model Builder** window, under **Results**, Ctrl-click to select **Velocity (spf3)**, **Pressure (spf3)**, **Velocity (spf4)**, and **Pressure (spf4)**.
- 2 Right-click and choose **Group**.

Verification

In the **Settings** window for **Group**, type **Verification** in the **Label** text field.

Topology Optimization

In the **Model Builder** window, under **Results** right-click **Topology Optimization** and choose **Delete**.

Diodicity

Compute the diodicity for the component (without power loss in solid domains).

Global Evaluation 2

- 1 In the **Model Builder** window, right-click **Diodicity** and choose **Global Evaluation**.
- 2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Verification/Solution 2 (3) (sol2)**.
- 4 Click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 2 (comp2)>Definitions>Variables>Di - Ratio of pressure differences**.
- 5 In the **Diodicity** toolbar, click  **Evaluate**.

The diodicity is actually higher for the verification simulation, so optimization has found a good design topology.

