

# Piezoelectric Rate Gyroscope

## Introduction

This model shows how to analyze a tuning fork based piezoelectric rate gyroscope. The reverse piezoelectric effect is used to drive an in-plane tuning fork mode. This mode is coupled to an out of plane mode by the Coriolis force and the resulting out of plane motion is sensed by the direct piezoelectric effect. The geometry of the tuning forks is designed so that the eigenfrequencies of the nearby modes are separated in frequency space. The frequency response of the system is computed and the rotation rate sensitivity is evaluated. Note that the model focuses on the performance of the sensor in a uniformly rotating reference frame. The model is based on the detailed analysis of a similar device presented in Ref. 1.

## Model Definition

Figure 1 shows the geometry of the device indicating the key features.



Figure 1: Device geometry showing the plane of symmetry through the center of the device and the key components of the gyroscope.

The packaging and fabrication of the device is discussed in Ref. 1. Here we provide a simple explanation of its principle of operation when operated in a rotating frame with no angular acceleration (Ref. 1 discusses the effects of an angular acceleration on the frequency response of the device in more detail). The gyroscope can be thought of as two

tuning forks, coupled together by a suspension structure. The suspension is anchored to the package of the device which is in turn attached to the rotating object. The drive tines are driven close to their resonance in an in-plane mode, as shown in Figure 2. The sense tines are designed to have a resonance at a nearby, but distinct, frequency with a significant out of plane component to their motion, as shown in Figure 3. As the drive mode vibrates in the in-plane direction within the rotating frame a Coriolis body force acts on the structure which excites the out of plane sense mode. The Coriolis force ( $\mathbf{F}_{cor}$ ) is given by:

$$\mathbf{F}_{\rm cor} = -2\rho\Omega \times \frac{\partial \mathbf{u}}{\partial t}$$

where  $\rho$  is the density of the material,  $\Omega$  is the angular velocity of the rotating frame and **u** is the local displacement of the structure. From the above equation it is clear that the Coriolis force is maximal when the angular velocity of the frame is parallel to the long inplane axis of the gyroscope structure. In this case the resulting force is in the out of plane direction and produces a corresponding out of plane motion of the drive tines. This motion causes reaction moments in the supporting suspension which in turn transfers these moments to the sense times — driving the sense mode. Note that in this model the angular velocity vector is assumed to be parallel to the long axis of the device.

The tines are fabricated from single crystal quartz wafers with the crystallographic Z-axis aligned parallel to the normal of the wafer plane. The details of the design are discussed in Ref. 1, but the critical point is that the electrodes are patterned in such a way that both inplane and in-phase out-of-plane motion of the sense tines is not detected by the sense electrodes. This leads to the rejection of unwanted signals in the output of the sensor.

In general, for resonant structures like this model, a very fine mesh is required to achieve accurate frequency response results. In the interest of saving time, we choose to use a relatively coarse mesh for this tutorial. As a result the resonant peak will shift if a more refined mesh is used instead.

## Results and Discussion

Figure 2 shows the eigenmode corresponding to the drive mode and Figure 3 shows that corresponding to the sense mode. Both the in-plane and out-of-plane motions of these modes are shown separately in the figures.



Figure 2: Drive mode, showing both in-plane motion (right) and out-of-plane motion (left). Note that the amplitude scale is arbitrary — only the relative value of the in-plane and outof-plane displacements has physical significance.



Eigenfrequency=10630+0.26472i Hz Surface: Displacement field, X component (mm) Surface: Displacement field, Z component (mm)

Figure 3: Sense mode, showing out-of-plane motion (left) and in-plane motion (right). Note that the amplitude scale is arbitrary — only the relative value of the in-plane and out-of-plane displacements has physical significance.



Figure 4: Sense voltage vs. drive frequency with an applied sinusoidal drive voltage of amplitude 2 V and an angular acceleration of 64 deg/s.



Figure 5: Sense voltage vs. angular acceleration at a drive voltage amplitude of 2 V and a frequency of 8396 Hz.

Figure 4 shows the response of the device as the frequency of the drive voltage waveform is varied. A clear peak in the response close to the drive frequency, at approximately 8396 Hz is apparent. This is the optimum drive frequency for the device. Figure 5 shows the sense voltage against the angular acceleration with a 2 V drive voltage at a frequency close to this optimum. As expected the response of the sensor is linear in this operation range, with a sensitivity of approximately 0.015 mV /(deg/s).

## Reference

1. S.D. Senturia, "A Piezoelectric Rate Gyroscope," *Microsystem Design*, chapter 21, Springer, 2000.

**Application Library path:** MEMS\_Module/Piezoelectric\_Devices/ piezoelectric\_rate\_gyroscope

## Modeling Instructions

From the File menu, choose New.

## NEW

In the New window, click 🔗 Model Wizard.

## MODEL WIZARD

- I In the Model Wizard window, click 间 3D.
- 2 In the Select Physics tree, select Structural Mechanics>Electromagnetics-Structure Interaction>Piezoelectricity>Piezoelectricity, Solid.
- 3 Click Add.
- 4 Click 🔿 Study.
- 5 In the Select Study tree, select Preset Studies for Selected Multiphysics>Eigenfrequency.
- 6 Click 🗹 Done.

## GEOMETRY I

Add some global parameters.

#### 6 | PIEZOELECTRIC RATE GYROSCOPE

#### **GLOBAL DEFINITIONS**

## Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.

**3** In the table, enter the following settings:

Name	Expression	Value	Description
va	64[deg/s]	I.II7 rad/s	Rotation angular velocity

Build a symmetric mesh for this model so that the numerical result for no rotation will be very close to the expected null result. To prevent the symmetry of the mesh from being broken, clear the **Avoid inverted elements by curving interior domain elements** check box.

## COMPONENT I (COMPI)

- I In the Model Builder window, click Component I (compl).
- 2 In the Settings window for Component, locate the Curved Mesh Elements section.
- **3** Clear the Avoid inverted elements by curving interior domain elements check box.

Import the geometry from file.

#### GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.
- 4 In the Geometry toolbar, click Insert Sequence and choose Insert Sequence.
- 5 Browse to the model's Application Libraries folder and double-click the file piezoelectric\_rate\_gyroscope\_geom\_sequence.mph.
- 6 In the Geometry toolbar, click 🟢 Build All.
- 7 Click the 4 Zoom Extents button in the Graphics toolbar.

The **Adaptive Frequency Sweep** study step will generate a high resolution frequency sweep. To avoid large file size, create an "explicit selection" to store solution data only on the external surfaces of the modeling domain.

## DEFINITIONS

External surfaces

I In the **Definitions** toolbar, click **here Explicit**.

- 2 In the Settings window for Explicit, locate the Input Entities section.
- **3** Select the **All domains** check box.
- 4 Locate the **Output Entities** section. From the **Output entities** list, choose **Adjacent boundaries**.
- 5 In the Label text field, type External surfaces.

Add the built-in quartz material.

## ADD MATERIAL

- I In the Home toolbar, click 🙀 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Piezoelectric>Quartz LH (1978 IEEE).
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

Set up the physics.

## SOLID MECHANICS (SOLID)

Piezoelectric Material I

In the Model Builder window, under Component I (comp1)>Solid Mechanics (solid) click Piezoelectric Material I.

## Mechanical Damping 1

- I In the Physics toolbar, click 📃 Attributes and choose Mechanical Damping.
- 2 In the Settings window for Mechanical Damping, locate the Damping Settings section.
- 3 From the Damping type list, choose Isotropic loss factor.
- **4** From the  $\eta_s$  list, choose **User defined**. In the associated text field, type 5e-5.

Anchor the circular region underneath the structure.

#### Fixed Constraint I

- I In the Physics toolbar, click 🔚 Boundaries and choose Fixed Constraint.
- 2 Select Boundaries 48 and 60 only.

Add rotating frame physics.

#### Rotating Frame 1

- I In the Physics toolbar, click 📄 Domains and choose Rotating Frame.
- 2 In the Settings window for Rotating Frame, locate the Rotating Frame section.

- 3 From the Axis of rotation list, choose y-axis.
- **4** In the  $\Omega$  text field, type va.
- 5 Locate the Frame Acceleration Effect section. Select the Coriolis force check box.

## **ELECTROSTATICS (ES)**

Add boundary conditions for the drive and sense electrodes.

I In the Model Builder window, under Component I (compl) click Electrostatics (es).

Drive Terminal 1

- I In the Physics toolbar, click 🔚 Boundaries and choose Terminal.
- 2 In the Settings window for Terminal, type Drive Terminal 1 in the Label text field.
- **3** Select Boundaries 28, 29, 85, 86, 101, and 102 only.
- 4 Locate the Terminal section. From the Terminal type list, choose Voltage.

## Drive Terminal 2

- I In the Physics toolbar, click 🔚 Boundaries and choose Terminal.
- 2 In the Settings window for Terminal, type Drive Terminal 2 in the Label text field.
- **3** Select Boundaries 24, 25, 46, 47, 89, and 90 only.
- 4 Locate the Terminal section. From the Terminal type list, choose Voltage.
- **5** In the  $V_0$  text field, type -1.

## Sense Terminal I

- I In the Physics toolbar, click 🔚 Boundaries and choose Terminal.
- 2 In the Settings window for Terminal, type Sense Terminal 1 in the Label text field.
- **3** Select Boundaries 21, 32, 79, and 94 only.

## Sense Terminal 2

- I In the Physics toolbar, click 🔚 Boundaries and choose Terminal.
- 2 In the Settings window for Terminal, type Sense Terminal 2 in the Label text field.
- 3 Select Boundaries 20, 33, 80, and 93 only.

Build a symmetric mesh for this model so that the numerical result for no rotation will be very close to the expected null result. To save computation time and to reduce file size, a relatively coarse mesh is used.

## MESH I

I In the Model Builder window, under Component I (compl) click Mesh I.

- 2 In the Settings window for Mesh, locate the Sequence Type section.
- 3 From the list, choose User-controlled mesh.

## Size

- I In the Model Builder window, under Component I (compl)>Mesh I click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- **3** From the **Predefined** list, choose **Finer**.
- 4 Click the **Custom** button.
- **5** Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type tQz/4.
- **6** In the **Minimum element size** text field, type tQz/12.

Free Tetrahedral I

- I In the Model Builder window, click Free Tetrahedral I.
- 2 In the Settings window for Free Tetrahedral, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domain 1 only.

#### Copy Domain I

- I In the Model Builder window, right-click Mesh I and choose Copying Operations> Copy Domain.
- **2** Select Domain 1 only.
- 3 In the Settings window for Copy Domain, locate the Destination Domains section.
- **4** Click to select the **Delta Activate Selection** toggle button.
- **5** Select Domains 2–4 only.

6 In the Home toolbar, click 📗 Build Mesh.



Set up the eigenfrequency study to solve for the eigenmodes.

## EIGENMODES

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Eigenmodes in the Label text field.

## Step 1: Eigenfrequency

- I In the Model Builder window, under Eigenmodes click Step I: Eigenfrequency.
- 2 In the Settings window for Eigenfrequency, locate the Study Settings section.
- 3 Select the Desired number of eigenfrequencies check box. In the associated text field, type9.
- 4 In the Search for eigenfrequencies around text field, type 3e3.
- 5 From the Eigenfrequency search method around shift list, choose Larger real part.
- 6 In the **Home** toolbar, click **= Compute**.

## RESULTS

#### Mode Shape (solid)

The default mode shape plot shows a surface plot of the displacement magnitude. To provide a deeper insight into the mode shapes, plot the x and z displacements in two separate surface plots instead.

Change the default surface plot to plot the x displacement.

## Surface 1

- I In the Model Builder window, expand the Mode Shape (solid) node, then click Surface I.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type u.

Duplicate the surface plot to plot the z displacement.

## Surface 2

- I Right-click Results>Mode Shape (solid)>Surface I and choose Duplicate.
- 2 In the Settings window for Surface, locate the Expression section.
- **3** In the **Expression** text field, type w.
- 4 Locate the Coloring and Style section. Click Change Color Table.
- 5 In the Color Table dialog box, select Rainbow>RainbowLight in the tree.
- 6 Click OK.

#### Deformation

- I In the Model Builder window, expand the Surface 2 node, then click Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- 3 In the x-component text field, type -w\_f\*1.3.
- 4 In the **y-component** text field, type 0.
- **5** In the **z-component** text field, type **0**.
- 6 Locate the Scale section.
- 7 Select the Scale factor check box. In the associated text field, type 1.

#### Mode Shape (solid)

Turn on color legend to see the relative amplitude of the x and z displacements.

- I In the Model Builder window, under Results click Mode Shape (solid).
- 2 In the Settings window for 3D Plot Group, locate the Color Legend section.
- 3 Select the Show legends check box.

4 From the **Position** list, choose **Alternating**.

Plot the mode shape of the drive mode.

- 5 Locate the Data section. From the Eigenfrequency (Hz) list, choose 8396.2+0.20848i.
- 6 In the Mode Shape (solid) toolbar, click 💽 Plot.
- 7 Click the **F** Zoom Extents button in the **Graphics** toolbar.

Plot the sense mode, which is at a higher frequency.

- 8 From the Eigenfrequency (Hz) list, choose 10630+0.26472i.
- 9 In the Mode Shape (solid) toolbar, click 💽 Plot.

**IO** Click the **Come Extents** button in the **Graphics** toolbar.

Set up and solve an **Adaptive Frequency Sweep** study, which is optimized for resolving narrow resonant peaks without excessive computation.

## ADD STUDY

- I In the Home toolbar, click  $\overset{\sim}{\sim}_1$  Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select Empty Study.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click  $\sim 2$  Add Study to close the Add Study window.

## FREQUENCY RESPONSE

- I In the Settings window for Study, type Frequency Response in the Label text field.
- 2 Locate the Study Settings section. Clear the Generate default plots check box.

## Adaptive Frequency Sweep

- I In the Study toolbar, click Study Steps and choose Frequency Domain> Adaptive Frequency Sweep.
- 2 In the Settings window for Adaptive Frequency Sweep, locate the Study Settings section.
- **3** In the **Frequencies** text field, type range(-1.3,0.02,1.3)+8396.2.
- 4 From the AWE expression type list, choose User controlled.
- **5** In the table, enter the following settings:

## Asymptotic waveform evaluation (AWE) expressions abs(comp1.es.V0\_4-comp1.es.V0\_3)/1[mV]

To reduce file size, only store solution data on the external surfaces.

- 6 Locate the Values of Dependent Variables section. Find the Store fields in output subsection. From the Settings list, choose For selections.
- 7 Under Selections, click + Add.
- 8 In the Add dialog box, select External surfaces in the Selections list.
- 9 Click OK.
- **IO** In the **Study** toolbar, click **= Compute**.

Create plots to visualize the frequency response of the displacement and the sense voltage.

## RESULTS

Frequency Response: Displacement

- I Right-click Mode Shape (solid) and choose Duplicate.
- 2 In the **Settings** window for **3D Plot Group**, type Frequency Response: Displacement in the **Label** text field.
- 3 Locate the Data section. From the Dataset list, choose Frequency Response/ Solution 2 (sol2).
- **4** In the Frequency Response: Displacement toolbar, click **I** Plot.

Frequency Response: Sense Voltage

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Frequency Response: Sense Voltage in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Frequency Response/ Solution 2 (sol2).

Sense Voltage

- I Right-click Frequency Response: Sense Voltage and choose Global.
- 2 In the Settings window for Global, type Sense Voltage in the Label text field.
- 3 Locate the y-Axis Data section. In the table, enter the following settings:

Expression	Unit	Description
abs(es.V0_4-es.V0_3)	mV	Sense Voltage

4 Click to expand the Coloring and Style section. From the Width list, choose 2.

## 5 In the Frequency Response: Sense Voltage toolbar, click 💽 Plot.

Add a study and make a plot to evaluate the sensitivity of the device.

#### ADD STUDY

- I In the Home toolbar, click 2 Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Frequency Domain.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click  $\stackrel{\sim}{\sim}_1$  Add Study to close the Add Study window.

## STUDY 3

- Step 1: Frequency Domain
- I In the Settings window for Frequency Domain, locate the Study Settings section.
- 2 In the Frequencies text field, type 8396.2.
- 3 Click to expand the Study Extensions section. Select the Auxiliary sweep check box.
- 4 Click + Add.
- **5** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
va (Rotation angular velocity)	0 32 64	deg/s

- 6 In the Model Builder window, click Study 3.
- 7 In the Settings window for Study, locate the Study Settings section.
- 8 Clear the Generate default plots check box.
- 9 In the Label text field, type Sensitivity.
- **IO** In the **Home** toolbar, click **= Compute**.

## RESULTS

Sensitivity: Sense Voltage vs. Angular Velocity

- I In the Model Builder window, right-click Frequency Response: Sense Voltage and choose Duplicate.
- 2 In the Settings window for ID Plot Group, locate the Data section.
- 3 From the Dataset list, choose Sensitivity/Solution 3 (sol3).
- 4 Locate the Legend section. From the Position list, choose Upper left.
- 5 In the Label text field, type Sensitivity: Sense Voltage vs. Angular Velocity.

Sense Voltage

- I In the Model Builder window, expand the Sensitivity: Sense Voltage vs. Angular Velocity node, then click Sense Voltage.
- 2 In the Settings window for Global, locate the x-Axis Data section.
- **3** From the **Unit** list, choose **deg/s**.
- 4 In the Sensitivity: Sense Voltage vs. Angular Velocity toolbar, click 💿 Plot.