



# Pull-In Voltage for a Biased Resonator — 2D

## *Introduction*

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Silicon micromechanical resonators have long been used for designing sensors and are now becoming increasingly important as oscillators in the consumer electronics market. In this sequence of models, a surface micromachined MEMS resonator, designed as part of a micromechanical filter, is analyzed in detail. The resonator is based on that developed in [Ref. 1](#).

This model performs a pull-in analysis of the structure, to predict the point at which the biased system becomes unstable. The analysis begins from the stationary analysis performed in the accompanying model [Stationary Analysis of a Biased Resonator — 2D](#); please review this model first.

## *Model Definition*

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The geometry, fabrication, and operation of the device are discussed for the “Stationary Analysis of a Biased Resonator” model.

This model computes the pull-in voltage for the resonator by solving an inverse problem. The y-coordinate of the resonator midpoint is computed using an integration operator (`intop1`). The inverse problem that COMSOL solves computes the DC voltage that must be applied to the beam in order to move the midpoint to a set y-coordinate, `yset`. This is achieved by adding a global equation for the DC voltage, `VdcSP`, applied to the resonator. The equation `intop1(y) - yset = 0` is solved to determine the value of `VdcSP`. This means that `VdcSP` is adjusted until the midpoint of the resonator has a y-coordinate given by the set value, `yset`. Essentially COMSOL is being asked to find the voltage that allows the beam to exist in equilibrium (stable or unstable) at a given displacement. Solving the problem in this manner avoids complications with trying to solve a problem with no solution (which is what happens if the voltage is continuously ramped up eventually exceeding the pull-in voltage). The result of the analysis is a displacement versus voltage plot, with a minimum at the pull-in voltage. Note that for a linear spring, the pull-in displacement corresponds to 1/3 of the gap distance. Although the inclusion of geometric nonlinearities in the solid mechanics solver means that the pull-in displacement changes slightly from this value, it is usually most efficient to search around this point for the pull-in voltage.

## Results and Discussion

Figure 1 shows the voltage-displacement curve for the resonator at equilibrium, for y-coordinates that correspond to displacements of around 1/3 of the gap size. The pull-in voltage is 63.3 V.

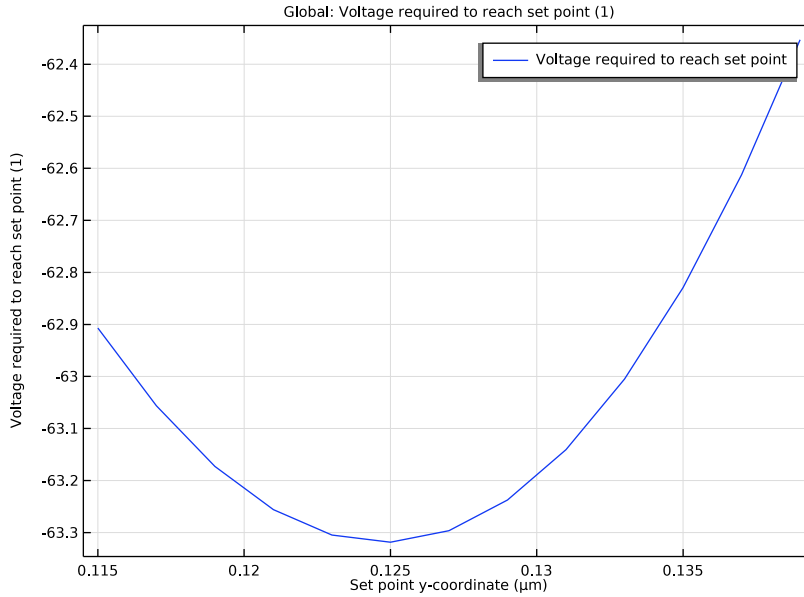
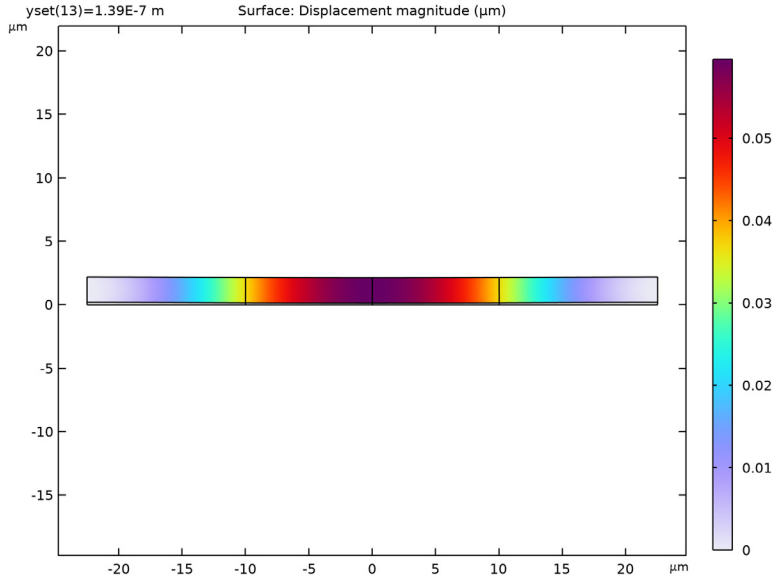


Figure 1: Voltage required to achieve a set displacement versus the target displacement. The pull-in voltage is the minimum of the plot: 63.3 V.



*Figure 2: y-displacement of the resonator at pull-in. The displacement at pull-in is 74 nm. For a linear spring the displacement at pull in would be 66 nm.*

Figure 2 shows the y-displacement of the resonator at the pull-in voltage. The maximum displacement at pull-in is 74 nm. This is comparable to the (approximate) linear spring value of 66 nm.

### *Notes About the COMSOL Implementation*

To compute the voltage required to generate the desired displacement of the beam, use a global equation. A common use of global equations is for computing the value of a dependent variable based on an ordinary differential equation in the dependent variable itself. However, it is also possible to couple a global equation with the other PDEs in the model as a powerful tool to solve certain kinds of inverse problems. This model uses a global equation to compute the potential applied to the drive electrode. The equation takes the form

$$y_0 = y_{\text{set}}$$

where  $y_0$  is the  $y$ -coordinate of the midpoint of the beam’s underside and  $y_{\text{set}}$  is the desired  $y$ -coordinate. COMSOL Multiphysics computes the voltage to satisfy the constraint implied by the above equation.

*Reference*

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1. F.D. Bannon III, J.R. Clark and C.T.-C. Nguyen, “High-Q HF Microelectromechanical Filters,” *IEEE Journal of Solid State Circuits*, vol. 35, no. 4, pp. 512–526, 2000.

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**Application Library path:** MEMS\_Module/Actuators/  
biased\_resonator\_2d\_pull\_in


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*Modeling Instructions*

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Start from the existing stationary model.

**APPLICATION LIBRARIES**

- 1 From the **File** menu, choose **Application Libraries**.
- 2 In the **Application Libraries** window, select **MEMS Module>Actuators>biased\_resonator\_2d\_basic** in the tree.
- 3 Click  **Open**.  
Add a parameter to set the  $y$ -coordinate of the midpoint of the resonator.

**GLOBAL DEFINITIONS**

*Parameters 1*


- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
yset	100[nm]	1E-7 m	Set point $y$ -coordinate

Add a nonlocal integration coupling to compute the actual displacement.

## DEFINITIONS


### *Integration 1 (intop1)*

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)** node.
- 2 Right-click **Component 1 (comp1)**>**Definitions** and choose **Nonlocal Couplings**>**Integration**.
- 3 In the **Settings** window for **Integration**, locate the **Source Selection** section.
- 4 From the **Geometric entity level** list, choose **Point**.
- 5 Click  **Paste Selection**.
- 6 In the **Paste Selection** dialog box, type 8 in the **Selection** text field.
- 7 Click **OK**.

Change the drive potential to the value  $V_{dcSP}$  - which will be solved for in a global equation.


## ELECTROSTATICS (ES)

### *Electric Potential 1*

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)**>**Electrostatics (es)** node, then click **Electric Potential 1**.
- 2 In the **Settings** window for **Electric Potential**, locate the **Electric Potential** section.
- 3 In the  $V_0$  text field, type  $V_{dcSP}$  [V].
- 4 Click the  **Show More Options** button in the **Model Builder** toolbar.
- 5 In the **Show More Options** dialog box, in the tree, select the check box for the node **Physics**>**Equation-Based Contributions**.
- 6 Click **OK**.

Add a global equation to compute the voltage for a given displacement,  $V_{dcSP}$ .



### *Global Equations 1*

- 1 In the **Physics** toolbar, click  **Global** and choose **Global Equations**.
- 2 In the **Settings** window for **Global Equations**, locate the **Global Equations** section.
- 3 In the table, enter the following settings:

Name	$f(u, ut, utt, t)$ (I)	Initial value ( $u_0$ ) (I)	Initial value ( $u_{t0}$ ) (I/s)	Description
$V_{dcSP}$	$(intop1(y) - yset) / yset$	0	0	

Set up a parametric sweep over the displacement set point,  $yset$ .


**ADD STUDY**

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.
- 4 Click **Add Study** in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

**PULL IN**

- 1 In the **Model Builder** window, right-click **Study 2** and choose **Rename**.
- 2 In the **Rename Study** dialog box, type Pull In in the **New label** text field.
- 3 Click **OK**.


*Step 1: Stationary*


- 1 In the **Model Builder** window, under **Pull In** click **Step 1: Stationary**.
- 2 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.
- 3 Select the **Auxiliary sweep** check box.
- 4 Click  **Add**.
- 5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
yset (Set point y-coordinate)	range ( 115[ nm ] , 2[ nm ] , 140[ nm ] )	m

The problem is highly nonlinear due to the presence of the global equation, so the solver settings need to be adjusted accordingly.

*Solution 2 (sol2)*

- 1 In the **Study** toolbar, click  **Show Default Solver**.
- 2 In the **Model Builder** window, expand the **Solution 2 (sol2)** node.
- 3 In the **Model Builder** window, expand the **Pull In>Solver Configurations>Solution 2 (sol2)>Stationary Solver 1** node.
- 4 Right-click **Pull In>Solver Configurations>Solution 2 (sol2)>Stationary Solver 1** and choose **Fully Coupled**.
- 5 In the **Settings** window for **Fully Coupled**, click to expand the **Method and Termination** section.
- 6 From the **Nonlinear method** list, choose **Automatic highly nonlinear (Newton)**.


7 In the **Study** toolbar, click  **Compute**.

## RESULTS

### *Pull In Displacement*

1 In the **Settings** window for **2D Plot Group**, type Pull In Displacement in the **Label** text field.

2 In the **Pull In Displacement** toolbar, click  **Plot**.

3 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Compare the resulting plot with [Figure 2](#).

Determine the pull-in voltage by plotting VdcSP versus yset.

### *Pull In Plot*

1 In the **Home** toolbar, click  **Add Plot Group** and choose **ID Plot Group**.

2 Right-click **ID Plot Group 7** and choose **Rename**.

3 In the **Rename ID Plot Group** dialog box, type Pull In Plot in the **New label** text field.

4 Click **OK**.

### *Global 1*

1 Right-click **Pull In Plot** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **Data** section.


3 From the **Dataset** list, choose **Pull In/Solution 2 (sol2)**.


4 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
VdcSP	1	Voltage required to reach set point

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type yset.

7 In the **Pull In Plot** toolbar, click  **Plot**.


8 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Compare the resulting plot with [Figure 1](#). The pull in voltage is the minimum of the curve: around 63.3 V at yset = 125 nm.


### *Streamline 1*

1 In the **Model Builder** window, expand the **Results>Electric Potential (es)** node, then click **Streamline 1**.




- 2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.
- 3 In the **Separating distance** text field, type 0.005.
- 4 In the **Electric Potential (es)** toolbar, click  **Plot**.


#### *Streamline I*

- 1 In the **Model Builder** window, expand the **Results>Electric Field Norm (es)** node, then click **Streamline I**.
- 2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.
- 3 In the **Separating distance** text field, type 0.005.
- 4 In the **Electric Field Norm (es)** toolbar, click  **Plot**.

#### *Streamline I*

- 1 In the **Model Builder** window, expand the **Results>Electric Potential (es) I** node, then click **Streamline I**.
- 2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.
- 3 In the **Separating distance** text field, type 0.005.
- 4 In the **Electric Potential (es) I** toolbar, click  **Plot**.

#### *Streamline I*

- 1 In the **Model Builder** window, expand the **Results>Electric Field Norm (es) I** node, then click **Streamline I**.
- 2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.
- 3 In the **Separating distance** text field, type 0.005.
- 4 In the **Electric Field Norm (es) I** toolbar, click  **Plot**.

