# Normal Modes of a Biased Resonator — 2D

## *Introduction*

Silicon micromechanical resonators have long been used for designing sensors and are now becoming increasingly important as oscillators in the consumer electronics market. In this sequence of models, a surface micromachined MEMS resonator, designed as part of a micromechanical filter, is analyzed in detail. The resonator is based on that developed in Ref. 1.

This model performs a modal analysis on the resonator, with and without an applied DC bias. The analysis begins from the stationary analysis performed in the accompanying model Stationary Analysis of a Biased Resonator — 2D; please review this model first.

## *Model Definition*

The geometry, fabrication, and operation of the device are discussed for the "Stationary Analysis of a Biased Resonator" model.

This model performs a modal analysis on the structure, with and without applied DC voltage biases of different magnitudes. The bias already exists as a parameter in the model so the prestressed eigenfrequency solver needs no adjustment to the physics settings. To compute the unbiased eigenfrequency, the solver settings are adjusted to solve only the structural mechanics problem.

## *Results and Discussion*

Figure 1 shows the mode shapes for the resonator under different bias conditions.

The mode shape does not change significantly with applied bias and the first three modes have the expected shapes for a clamped-clamped beam. The frequency of the fundamental is reduced significantly by the applied bias, an effect known as spring softening (the response of higher-order modes was not computed as a function of applied bias).

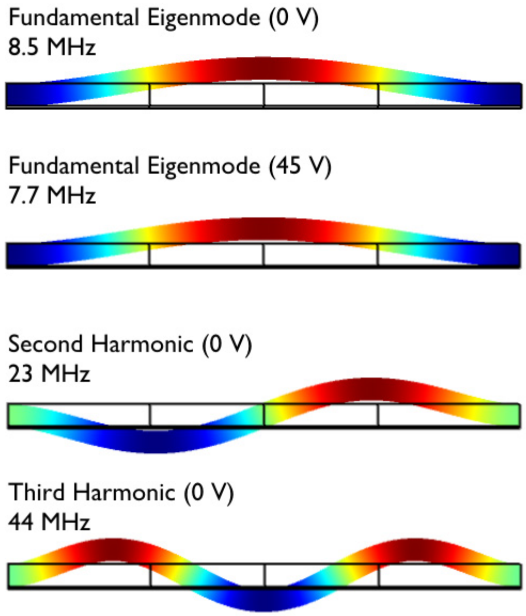


*Figure 1: Mode shapes for the resonator under different bias conditions . The mode frequencies are indicated in the figure. The colors visualize the relative y-displacement magnitude.*

The spring softening effect can be seen in detail in Figure 2. A clear decrease in the resonant frequency is evident with increasing bias voltage. This figure should be compared with Figure 16 of Ref. 1, where the same effect is apparent.

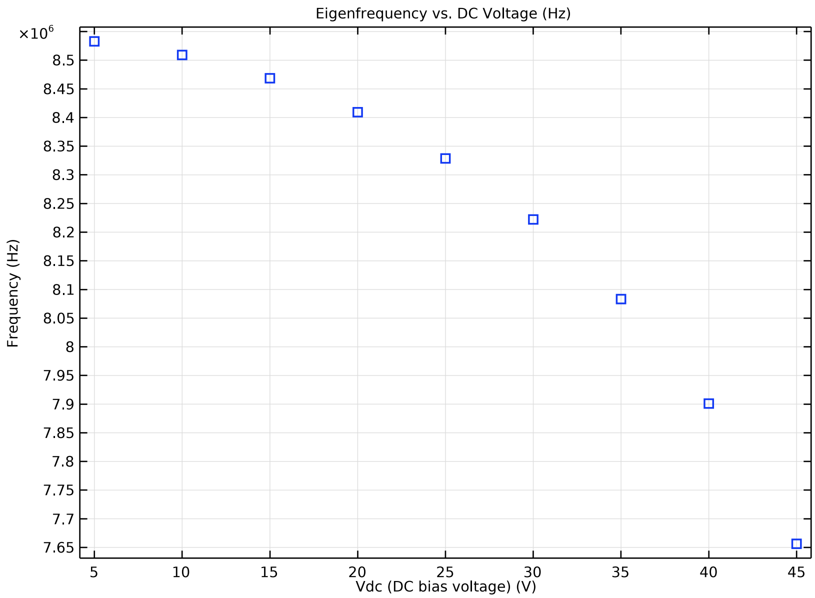


*Figure 2: Mode frequency shown against the applied DC voltage bias. The spring softening effect is evident. Compare with Fig. 16 of* Ref. 1.

## Notes About the COMSOL Implementation

This model excludes certain dependent variables from the solver settings in order to compute the unbiased eigenfrequency. By not computing for the electric potential or the displacement of the air domains, the model is equivalent to a pure solid mechanics problem, solved in the absence of external forces. Excluding dependent variables in the solver in this manner can be useful for debugging models as well as for computing uncoupled problems in this manner.

## Reference


1. F.D. Bannon III, J.R. Clark, and C. T.-C. Nguyen, “High-Q HF Microelectromechanical Filters”, *IEEE Journal of Solid State Circuits*, vol. 35, no. 4, pp. 512–526, 2000.

**Application Library path:** `MEMS_Module/Actuators/biased_resonator_2d_modes`

## Modeling Instructions

Start from the existing stationary model.

### APPLICATION LIBRARIES

1. From the **File** menu, choose **Application Libraries**.
2. In the **Application Libraries** window, select **MEMS Module>Actuators>biased_resonator_2d_basic** in the tree.
3. Click **Open**.

Add an unbiased eigenfrequency study. The settings for this study need to be modified so that only the structural part of the problem is solved.

### ADD STUDY

1. In the **Home** toolbar, click **Add Study** to open the **Add Study** window.
2. Go to the **Add Study** window.
3. Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Multiphysics>Eigenfrequency**.
4. Click **Add Study** in the window toolbar.
5. In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

### STUDY 2

#### *Step 1: Eigenfrequency*

1. In the **Settings** window for **Eigenfrequency**, locate the **Study Settings** section.
2. Select the **Desired number of eigenfrequencies** check box. In the associated text field, type `3`.
3. In the **Model Builder** window, click **Study 2**.
4. In the **Settings** window for **Study**, type `Unbiased Eigenfrequency` in the **Label** text field.

Set up the solver to solve only for the solid mechanics variables.

#### *Solution 2 (sol2)*

1. In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 2 (sol2)** node, then click **Dependent Variables 1**.

3 In the **Settings** window for **Dependent Variables**, locate the **General** section.

4 From the **Defined by study step** list, choose **User defined**.

5 In the **Model Builder** window, expand the **Unbiased Eigenfrequency> Solver Configurations>Solution 2 (sol2)>Dependent Variables 1** node, then click **Electric potential (comp1.V)**.

6 In the **Settings** window for **Field**, locate the **General** section.

7 Clear the **Solve for this field** check box.

8 Clear the **Store in output** check box.

9 In the **Model Builder** window, under **Unbiased Eigenfrequency>Solver Configurations> Solution 2 (sol2)>Dependent Variables 1** click **Spatial mesh displacement (comp1.spatial.disp)**.

10 In the **Settings** window for **Field**, locate the **General** section.

11 Clear the **Solve for this field** check box.

12 Clear the **Store in output** check box.

13 In the **Model Builder** window, click **Unbiased Eigenfrequency**.

14 In the **Settings** window for **Study**, locate the **Study Settings** section.

15 Clear the **Generate default plots** check box.

16 In the **Study** toolbar, click **= Compute**.

Set the dataset to be in the material frame for postprocessing. This allows the use of the deformation plot attribute.

## RESULTS

### *Unbiased Eigenfrequency/Solution 2 (sol2)*

1 In the **Model Builder** window, expand the **Results>Datasets** node, then click **Unbiased Eigenfrequency/Solution 2 (sol2)**.

2 In the **Settings** window for **Solution**, locate the **Solution** section.

3 From the **Frame** list, choose **Material (X, Y, Z)**.

Plot the mode shapes.

### *Unbiased Modes*

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Unbiased Eigenfrequency/Solution 2 (sol2)**.

4 In the **Label** text field, type `Unbiased Modes`.

*Surface 1*

1 Right-click **Unbiased Modes** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `v`.

*Deformation 1*

1 Right-click **Surface 1** and choose **Deformation**.

2 In the **Unbiased Modes** toolbar, click **Plot**.

3 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the mode shapes with those shown in Figure 1 for all the modes computed. To switch between the modes click **Unbiased Modes** and choose a different value from the **Eigenfrequency** list.

### ROOT

Add a **Eigenfrequency, Prestressed** study.

### ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Solid Mechanics>Eigenfrequency, Prestressed**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

### BIASED EIGENFREQUENCY

1 In the **Model Builder** window, right-click **Study 3** and choose **Rename**.

2 In the **Rename Study** dialog box, type `Biased Eigenfrequency` in the **New label** text field.

3 Click **OK**.

Create a parametric sweep over DC bias voltage.

*Parametric Sweep*

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 Click **Range**.

5 In the **Range** dialog box, type `5` in the **Start** text field.

6 In the **Step** text field, type `5`.

7 In the **Stop** text field, type `45`.

8 Click **Add**.

Solve for only the first eigenfrequency.

### *Step 2: Eigenfrequency*

1 In the **Model Builder** window, click **Step 2: Eigenfrequency**.

2 In the **Settings** window for **Eigenfrequency**, locate the **Study Settings** section.

3 Select the **Desired number of eigenfrequencies** check box. In the associated text field, type `1`.

Disable the default plots.

4 In the **Model Builder** window, click **Biased Eigenfrequency**.

5 In the **Settings** window for **Study**, locate the **Study Settings** section.

6 Clear the **Generate default plots** check box.

7 In the **Study** toolbar, click **Compute**.

## RESULTS

### *Biased Eigenfrequency/Parametric Solutions 1 (sol5)*

1 In the **Model Builder** window, under **Results>Datasets** click **Biased Eigenfrequency/ Parametric Solutions 1 (sol5)**.

2 In the **Settings** window for **Solution**, locate the **Solution** section.

3 From the **Frame** list, choose **Material (X, Y, Z)**.

### *Biased Modes*

1 In the **Model Builder** window, right-click **Unbiased Modes** and choose **Duplicate**.

2 In the **Settings** window for **2D Plot Group**, type `Biased Modes` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Biased Eigenfrequency/ Parametric Solutions 1 (sol5)**.

4 In the **Biased Modes** toolbar, click **Plot**.

**5** Click the **Zoom Extents** button in the **Graphics** toolbar.

Confirm the mode shape is similar to the unbiased fundamental mode.

Create a plot of eigenfrequency versus applied DC voltage.

*Eigenfrequency vs. DC voltage*

**1** In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

**2** In the **Settings** window for **1D Plot Group**, locate the **Data** section.

**3** From the **Dataset** list, choose **Biased Eigenfrequency/Parametric Solutions 1 (sol5)**.

**4** In the **Label** text field, type `Eigenfrequency vs. DC voltage`.

*Point Graph 1*

**1** Right-click **Eigenfrequency vs. DC voltage** and choose **Point Graph**.

**2** In the **Settings** window for **Point Graph**, locate the **Selection** section.

**3** Click **Paste Selection**.

**4** In the **Paste Selection** dialog box, type `1` in the **Selection** text field.

**5** Click **OK**.

**6** In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.

**7** In the **Expression** text field, type `solid.freq`.

**8** Locate the **x-Axis Data** section. From the **Axis source data** list, choose **Outer solutions**.

**9** Click to expand the **Title** section. From the **Title type** list, choose **Custom**.

**10** Find the **Type and data** subsection. Clear the **Type** check box.

**11** Clear the **Description** check box.

**12** Find the **User** subsection. In the **Prefix** text field, type `Eigenfrequency vs. DC Voltage`.

**13** Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

**14** Find the **Line markers** subsection. From the **Marker** list, choose **Square**.

Compare this plot with that in Figure 2. Note the spring softening effect.

**15** In the **Eigenfrequency vs. DC voltage** toolbar, click **Plot**.

**16** Click the **Zoom Extents** button in the **Graphics** toolbar.

*Streamline 1*

**1** In the **Model Builder** window, expand the **Results>Electric Potential (es)** node, then click **Streamline 1**.

**2** In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.

**3** In the **Separating distance** text field, type `0.005`.

**4** In the **Electric Potential (es)** toolbar, click **Plot**.

*Streamline 1*

**1** In the **Model Builder** window, expand the **Results>Electric Field Norm (es)** node, then click **Streamline 1**.

**2** In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.

**3** In the **Separating distance** text field, type `0.005`.

**4** In the **Electric Field Norm (es)** toolbar, click **Plot**.