



# Walking Instability in a Washing Machine

## *Introduction*

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Walking instability, due to nonuniform distribution of clothes, is a common problem in lightweight portable washing machines. This problem is more severe in horizontal-axis washing machines, which are more popular because of their high efficiency in spite of high manufacturing cost.

This model simulates a simplified model of a portable horizontal-axis washing machine and predicts the onset of walking instability during the spinning cycle. A control-based active balancing method is also implemented to eliminate the instability and vibrations present in the system.

## *Model Definitions*

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### **ABOUT THE INSTABILITY**

The instability occurs due to the presence of an unbalanced laundry mass during spinning. A significant centrifugal force is generated by the rotating unbalanced laundry mass which tends to destabilize the machine. This problem of instability used to be solved by adding additional weight to the machine. Nowadays there are better ways available to stabilize the system.

The instability could give rise to translational slip, rotational slip, or tip. It can be proved that the critical speed for impending translational slip is higher than that of rotational slip. Hence this model analyzes the rotational instability.

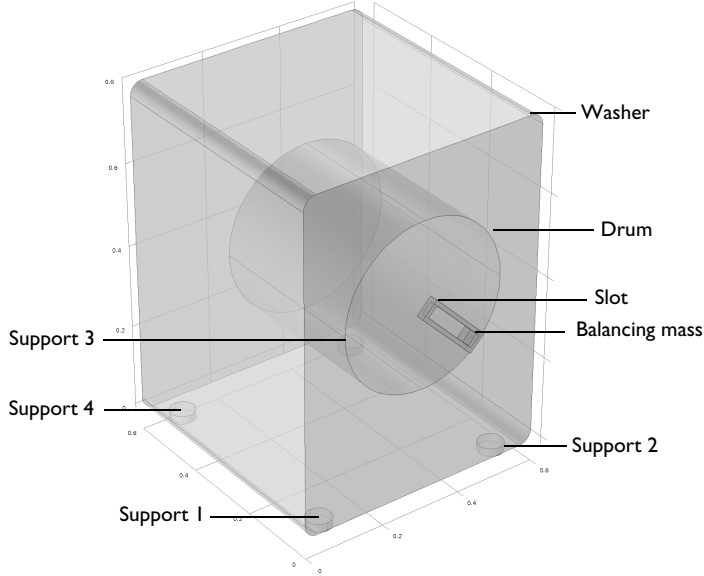
### **MODELING ASSUMPTIONS**

There are several assumptions taken to simplify the model:

- The drum and washer are assumed to be rigid.
- The drum is mounted to the washer so that the only relative motion is a rotation around the axis of the drum.
- The RPM of the drum is constant and high enough so that the laundry mass, due to centrifugal forces, rotates with the same speed as the drum.
- The machine remains in contact with the floor which means tip is not allowed.
- Friction between the washer and the ground is modeled using Coulomb friction model with a constant friction coefficient.

## GEOMETRY

The modeled geometry, consisting of an assembly of washer, drum, slot, and balancing mass, shown in [Figure 1](#).



*Figure 1: Modeled geometry of a washing machine.*

Ideally, the balancing mass should be placed on both the sides, front and back, of the drum to minimize the moment imbalance. However, for the ease of modeling, it is modeled only on the front side of the drum but its  $y$ -coordinate of the center of mass is shifted in such way that it coincides with that of the drum.

The drum and slot are assumed to have a negligible mass compared to the mass of the washer however the drum adds the mass of unbalanced clothes.

## CONNECTIONS

The drum is connected to the washer through a hinge joint. The drum and slot are also connected through a hinge joint. This joint is needed for active balancing and is locked when there is no balancing in the system. In case of active balancing, a corrective angular velocity is applied on this joint.

The slot and balancing mass are connected through a prismatic joint. This joint is also needed for active balancing. However in this model, it is not used and always locked.

The washer has four supports to the ground which are modeled with the planar joints. These planar joints have elasticity so that the joint forces can be computed independently on all the joints in spite of the washer (a rigid body) being connected to the ground at four different locations.

### SLIP MARGIN

Slip margin is a measure of stability and walk occurs when it goes to zero. It is defined as the difference between the maximum possible friction force and the actual friction force.

$$M_{sl} = |\mu F_z| - \sqrt{F_x^2 + F_y^2}$$

Slip margin is defined for all the supports of the machine and the machine actually slips when at least three or all four supports have zero slip margin. The slip margin is a function of the angular speed of the drum and hence it defines the critical operational speed to avoid the machine start walking.

In practice, the active balancing system should be activated when this slip margin is close to zero. In this model, the drum's angular speed is known and linearly increased with time, and the balancing system is activated at a certain time.

### CONTROL MECHANISM FOR ACTIVE BALANCING

The cause of imbalance is the net unbalanced centrifugal force on the rotating clothes. If an equal and opposite force is applied, the vibrations can be eliminated. This can be achieved with the help of a balancing mass. In general, the mass and position of unbalanced clothes are not known a priori and the mass of balancing mass is constant. The angular and radial position of the balancing mass must thus be adjusted in order to balance the forces created by the unbalanced clothes by the balancing mass.

The control mechanism for active balancing performs two type of corrections:

- Angular correction: the rotation of the slot-balancing mass assembly with respect to the drum. This is to correct the direction of the centrifugal force generated by the balancing mass.
- Radial correction: the translation of the balancing mass in the slot. This is to correct the magnitude of the centrifugal force generated by the balancing mass.

In this model, parameters are chosen so that radial correction is not required and hence it is not modeled. However the model can be extended to also implement this correction.

#### *Angular correction*

The total imbalance in the  $xz$ -plane in the rotating frame can be written as:

$$Fx_l = m_b r_b \omega^2 \cos(\phi_0 + \beta), \quad Fz_l = eq m_{cl} r_{cl} \omega^2 - m_b r_b \omega^2 \sin(\phi_0 + \beta)$$

where

- $m_b$  is the mass of the balancing mass
- $m_{cl}$  is the mass of the unbalanced clothes
- $r_b$  is the radial position of the balancing mass
- $r_{cl}$  is the radial position of the unbalanced clothes
- $\omega$  is the angular velocity of the drum
- $\phi_0$  is the initial angle of the balancing mass with horizontal axis
- $\beta$  is the relative rotation between the drum and the slot

The total imbalance magnitude and the angle between the total imbalance and the balancing mass is given as:

$$F_{tot} = \sqrt{Fx_l^2 + Fz_l^2}, \quad \theta = \phi_0 + \beta + \text{atan}\left(\frac{Fz_l}{Fx_l}\right)$$

The correction angle can be computed as:

$$d\theta = \text{atan}\left(\frac{F_{tot} \sin \theta}{m_b r_b \omega^2 - F_{tot} \cos \theta}\right)$$

## Results and Discussion

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[Figure 2](#) shows the rotation of the washer at a particular instant for a system without active balancing. Here displacement is scaled by a factor of 100 for better visualization.

The direction and magnitude of the friction force on all the supports at a particular instant can be seen in [Figure 3](#).

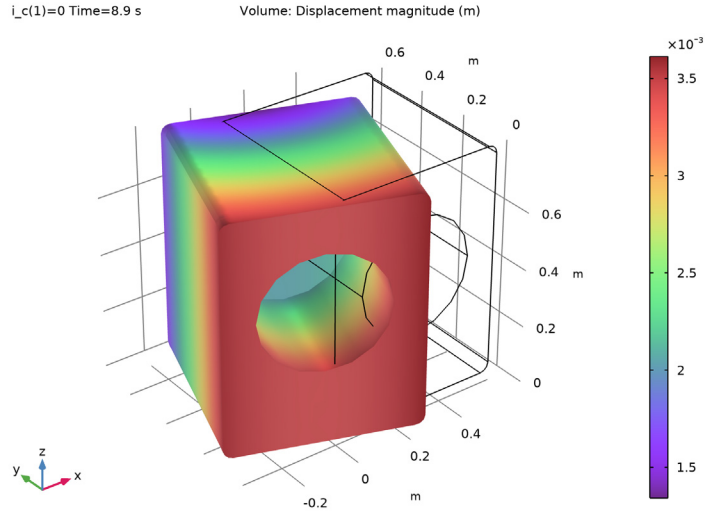


Figure 2: Washer rotation for a system without active balancing (magnified).

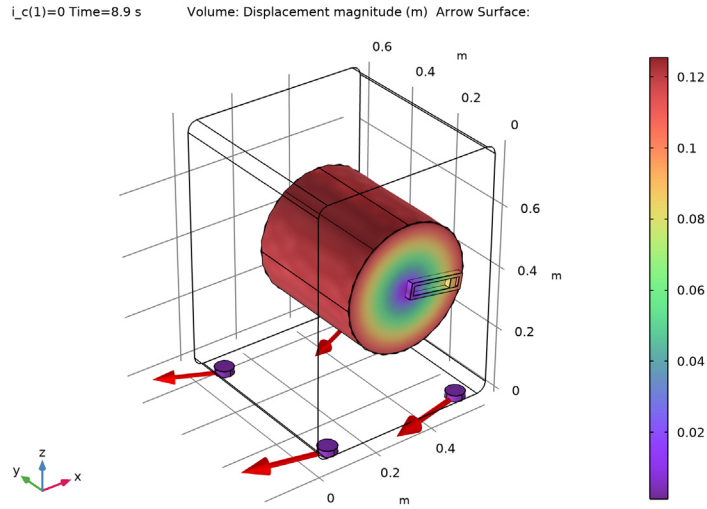


Figure 3: Drum rotation and friction force at washer supports for a system without active balancing.

Figure 4 shows the total imbalance in the rotating frame. The total imbalance increases with the increase in drum angular velocity. The effect of active balancing is clearly visible. It reduces the imbalance to a very small value as soon as it is activated. The variation of total imbalance in the fixed frame is shown in Figure 5.

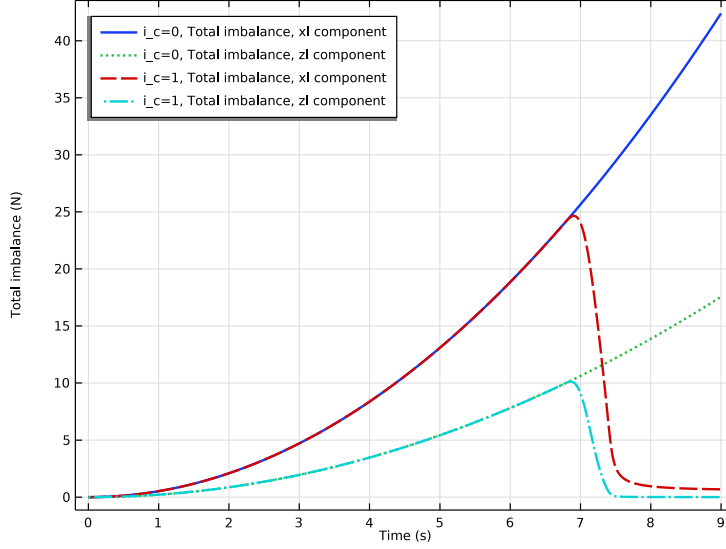


Figure 4: Total imbalance in the rotating frame.

The time variation of slip margin for support-1 (front) and support-3 (back) are shown in Figure 6. It can be observed that the slip margin on support-3 is higher than that of support-1. This indicates that in this particular design, the front support is more likely to slip.

The total slip margin of the washing machine can be seen in Figure 7. Total slip margin is the sum of slip margins of individual supports. It can be seen in the plot that the total slip margin becomes zero for short durations which gives rise to the slip of the washing machine. In case of active balancing, the total slip margin is improved as soon as the balancing mechanism is activated.

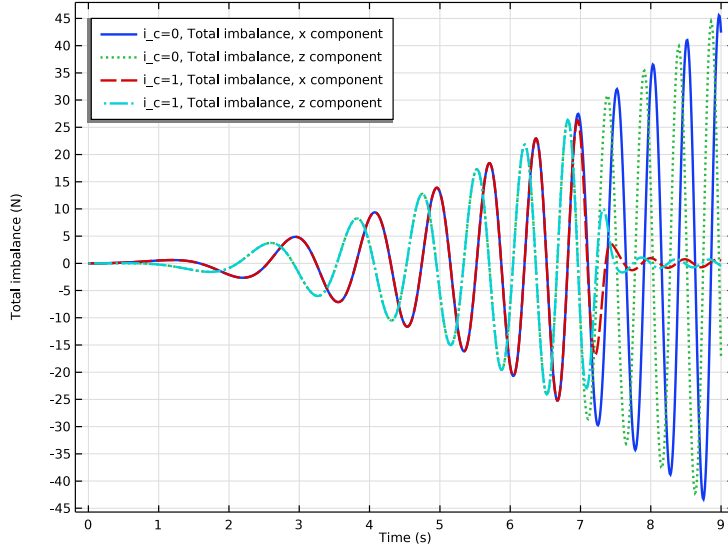


Figure 5: Total imbalance in the fixed frame.

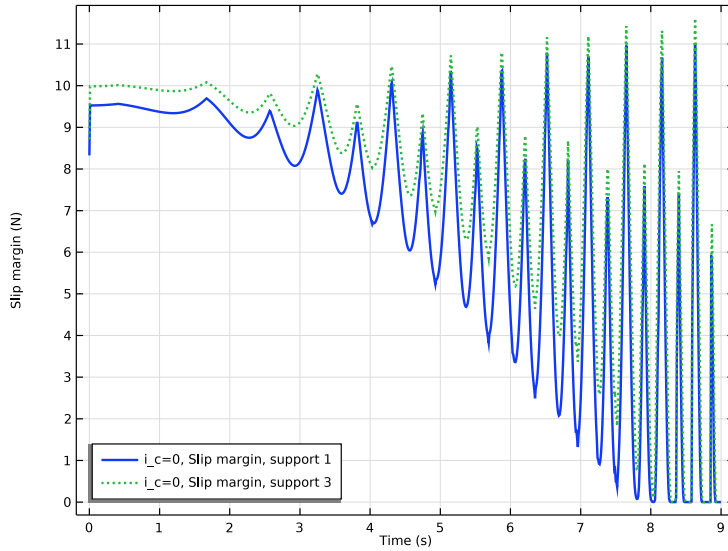
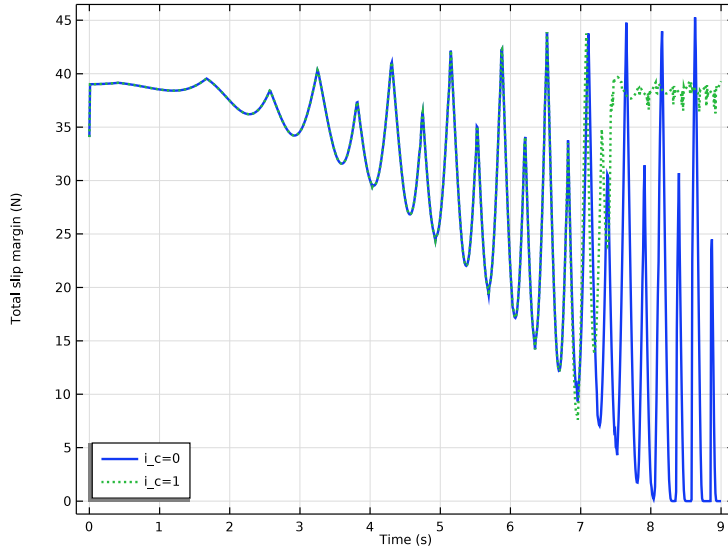


Figure 6: Slip margins of support-1 (front) and support-3 (back) for a system without active balancing.



*Figure 7: Total slip margin of the washing machine.*

The rotation of the washer about  $z$ -axis is shown in [Figure 8](#). It shows the rotational instability developing with time which is eliminated once the balancing mechanism is activated.

[Figure 9](#) shows the rpm of the drum and the correction motor. It can be seen that correction motor starts working around 7 seconds and stops as soon as the system is stabilized. The correction angle needed to stabilize the system is shown in [Figure 10](#).

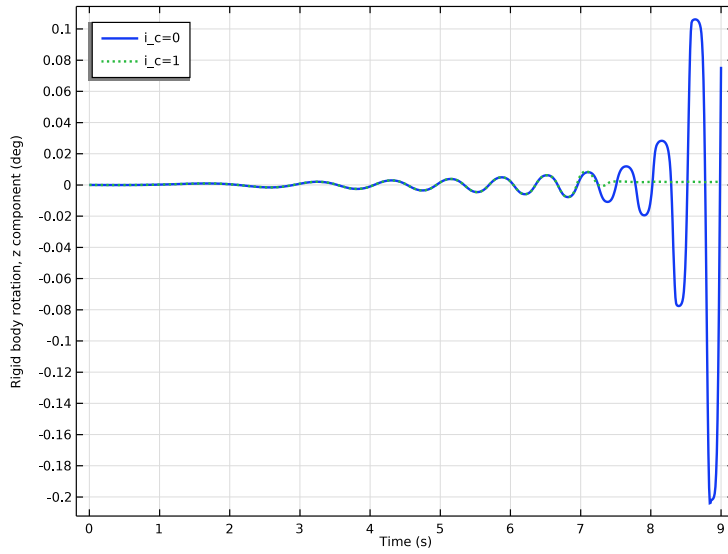


Figure 8: Rotation of washer about vertical axis.

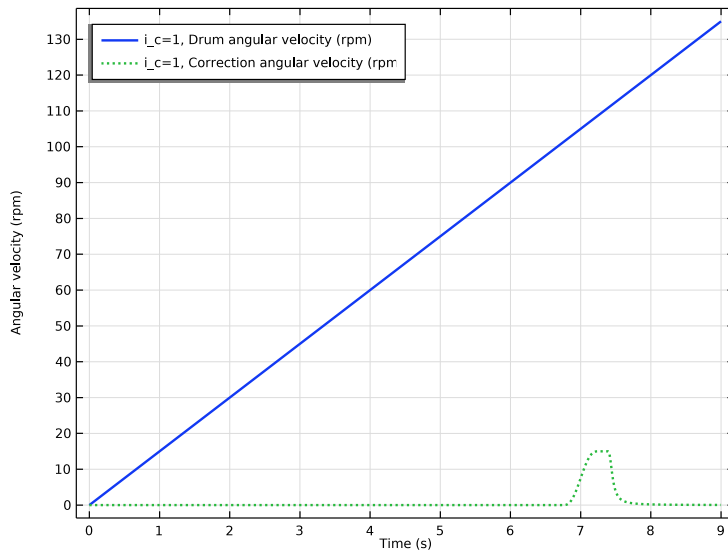


Figure 9: RPM of the drum and the correction motor.

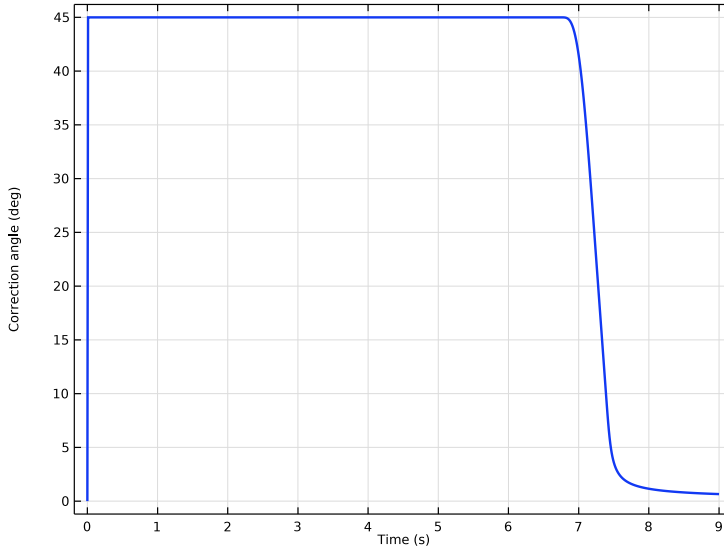


Figure 10: Required correction angle for a system with active balancing.

### Notes About the COMSOL Implementation

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- The **Mass and Moment of Inertia** subnode of the **Rigid Material** is used to enter the inertia properties given at a certain point.
- The connection between the washer and the ground is modeled using a **Planar Joint**. The rigid washer is in touch with the ground at four places. To apply friction on these supports, the joint forces should be known. To compute the joint forces and to eliminate the additional constraints, joints are made elastic with some high artificial spring and damping coefficients.
- The information about system degrees of freedom and system constraints can be found in the Rigid Body DOF Summary section of the physics node.
- The connections set up in the model can also be reviewed in the Joints Summary section of the physics node.

### References

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1. E. Papadopoulos and I. Papadimitriou, “Modeling, Design and Control of a Portable Washing Machine During the Spinning Cycle,” *IEEE-ASME International Conference on Advanced Intelligent Mechatronics Systems*, vol. 2, pp. 899–904, 2001.

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**Application Library path:** Multibody\_Dynamics\_Module/  
Machinery\_and\_Robotics/washing\_machine\_walk

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### *Modeling Instructions*

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From the **File** menu, choose **New**.

#### **NEW**

In the **New** window, click  **Model Wizard**.

#### **MODEL WIZARD**

- 1 In the **Model Wizard** window, click  **3D**.
- 2 In the **Select Physics** tree, select **Structural Mechanics>Multibody Dynamics (mbd)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **General Studies>Time Dependent**.
- 6 Click  **Done**.

#### **GLOBAL DEFINITIONS**

Start by importing the model parameters and geometry.

##### *Parameters I*

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters I**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 Click  **Load from File**.
- 4 Browse to the model's Application Libraries folder and double-click the file `washing_machine_walk_parameters.txt`.

#### **GEOMETRY I**

##### *Import I (impl)*

- 1 In the **Home** toolbar, click  **Import**.
- 2 In the **Settings** window for **Import**, locate the **Import** section.
- 3 Click  **Browse**.

- 4 Browse to the model's Application Libraries folder and double-click the file washing\_machine\_walk.mphbin.
- 5 Click  **Import**.

#### *Form Union (fin)*

- 1 In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** click **Form Union (fin)**.
- 2 In the **Settings** window for **Form Union/Assembly**, locate the **Form Union/Assembly** section.
- 3 From the **Action** list, choose **Form an assembly**.
- 4 Clear the **Create pairs** check box.
- 5 In the **Home** toolbar, click  **Build All**.
- 6 Click the  **Go to Default View** button in the **Graphics** toolbar.
- 7 Click the  **Zoom Extents** button in the **Graphics** toolbar.

### **MULTIBODY DYNAMICS (MBD)**

#### *Rigid Material: Washer*

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Multibody Dynamics (mbd)** and choose **Material Models>Rigid Material**.
- 2 In the **Settings** window for **Rigid Material**, type Rigid Material: Washer in the **Label** text field.
- 3 Select Domains 1–3, 7, and 8 only.  
Set the density of the selected rigid domains to zero. Use the **Mass and Moment of Inertia** subnode instead to specify the mass and center of mass of the domains.
- 4 Locate the **Density** section. From the  $\rho$  list, choose **User defined**.

#### *Mass and Moment of Inertia I*

- 1 In the **Physics** toolbar, click  **Attributes** and choose **Mass and Moment of Inertia**.
- 2 In the **Settings** window for **Mass and Moment of Inertia**, locate the **Center of Mass** section.
- 3 From the list, choose **User defined**.
- 4 Specify the  $\mathbf{X}_m$  vector as

|       |   |
|-------|---|
| xcx_w | x |
| xcy_w | y |
| xcz_w | z |

- 5 Locate the **Mass and Moment of Inertia** section. In the  $m$  text field, type m\_w.

Use an **Applied Force** subnode to account for the gravitational force.

*Rigid Material: Washer*

In the **Model Builder** window, click **Rigid Material: Washer**.

*Applied Force 1*

- 1 In the **Physics** toolbar, click  **Attributes** and choose **Applied Force**.
- 2 In the **Settings** window for **Applied Force**, locate the **Location** section.
- 3 From the list, choose **User defined**.
- 4 Specify the  $\mathbf{X}_p$  vector as

|       |   |
|-------|---|
| xcx_w | x |
| xcy_w | y |
| xcz_w | z |

- 5 Locate the **Applied Force** section. Specify the  $\mathbf{F}$  vector as

|              |   |
|--------------|---|
| 0            | x |
| 0            | y |
| -m_w*g_const | z |

Create more parts of the assembly by duplicating the **Rigid Material: Washer** node.

*Rigid Material: Drum*

- 1 Right-click **Rigid Material: Washer** and choose **Duplicate**.
- 2 In the **Settings** window for **Rigid Material**, type Rigid Material: Drum in the **Label** text field.
- 3 Locate the **Domain Selection** section. Click  **Clear Selection**.
- 4 Select Domain 4 only.

*Mass and Moment of Inertia 1*

- 1 In the **Model Builder** window, expand the **Rigid Material: Drum** node, then click **Mass and Moment of Inertia 1**.
- 2 In the **Settings** window for **Mass and Moment of Inertia**, locate the **Center of Mass** section.
- 3 Specify the  $\mathbf{X}_m$  vector as

|            |   |
|------------|---|
| xcx_d      | x |
| xcy_d      | y |
| xcz_d+r_cl | z |

4 Locate the **Mass and Moment of Inertia** section. In the  $m$  text field, type  $m\_cl$ .

*Applied Force I*

- 1 In the **Model Builder** window, click **Applied Force I**.
- 2 In the **Settings** window for **Applied Force**, locate the **Location** section.
- 3 Specify the  $\mathbf{X}_p$  vector as

|                |     |
|----------------|-----|
| $xcx\_d$       | $x$ |
| $xcy\_d$       | $y$ |
| $xcz\_d+r\_cl$ | $z$ |

4 Locate the **Applied Force** section. Specify the  $\mathbf{F}$  vector as

|                   |     |
|-------------------|-----|
| 0                 | $x$ |
| 0                 | $y$ |
| $-m\_cl*g\_const$ | $z$ |

*Rigid Material: Balancing mass*

- 1 In the **Model Builder** window, right-click **Rigid Material: Drum** and choose **Duplicate**.
- 2 In the **Settings** window for **Rigid Material**, type Rigid Material: Balancing mass in the **Label** text field.
- 3 Locate the **Domain Selection** section. Click  **Clear Selection**.
- 4 Select Domain 6 only.

*Mass and Moment of Inertia I*

- 1 In the **Model Builder** window, expand the **Rigid Material: Balancing mass** node, then click **Mass and Moment of Inertia I**.
- 2 In the **Settings** window for **Mass and Moment of Inertia**, locate the **Center of Mass** section.
- 3 Specify the  $\mathbf{X}_m$  vector as

|                          |     |
|--------------------------|-----|
| $xcx\_d+r\_b*\cos(phi0)$ | $x$ |
| $xcy\_d$                 | $y$ |
| $xcz\_d-r\_b*\sin(phi0)$ | $z$ |

4 Locate the **Mass and Moment of Inertia** section. In the  $m$  text field, type  $m\_b$ .

*Applied Force I*

- 1 In the **Model Builder** window, click **Applied Force I**.
- 2 In the **Settings** window for **Applied Force**, locate the **Location** section.

3 Specify the  $\mathbf{X}_p$  vector as

|                          |   |
|--------------------------|---|
| $xcx\_d+r\_b*\cos(phi0)$ | x |
| $xcy\_d$                 | y |
| $xcz\_d-r\_b*\sin(phi0)$ | z |

4 Locate the **Applied Force** section. Specify the  $\mathbf{F}$  vector as

|                  |   |
|------------------|---|
| 0                | x |
| 0                | y |
| $-m\_b*g\_const$ | z |

*Rigid Material: Slot*

- 1 In the **Physics** toolbar, click  **Domains** and choose **Rigid Material**.
- 2 In the **Settings** window for **Rigid Material**, type Rigid Material: Slot in the **Label** text field.
- 3 Select Domain 5 only.
- 4 Locate the **Density** section. From the  $\rho$  list, choose **User defined**.

Use planar joints to support the washer on the ground at various locations. Use the elastic version of the joint to reduce the system constraints and hence enabling the computation of joint forces.

*Planar Joint: Support 1*

- 1 In the **Physics** toolbar, click  **Global** and choose **Planar Joint**.
- 2 In the **Settings** window for **Planar Joint**, type Planar Joint: Support 1 in the **Label** text field.
- 3 Locate the **Attachment Selection** section. From the **Source** list, choose **Fixed**.
- 4 From the **Destination** list, choose **Rigid Material: Washer**.
- 5 Locate the **Axis of Joint** section. Specify the  $\mathbf{e}_0$  vector as

|   |   |
|---|---|
| 0 | x |
| 0 | y |
| 1 | z |

- 6 Locate the **Joint Elasticity** section. From the list, choose **Elastic joint**.
- 7 Locate the **Joint Forces and Moments** section. From the list, choose **Computed using weak constraints**.

#### *Center of Joint: Boundary 1*

- 1 In the **Model Builder** window, expand the **Planar Joint: Support 1** node, then click **Center of Joint: Boundary 1**.
- 2 Select Boundary 18 only.

#### *Joint Elasticity 1*

- 1 In the **Model Builder** window, click **Joint Elasticity 1**.
- 2 In the **Settings** window for **Joint Elasticity**, locate the **Spring** section.
- 3 In the  $k_u$  text field, type  $k_s$ .
- 4 In the  $k_\theta$  text field, type  $k_s$ .
- 5 Locate the **Viscous Damping** section. In the  $c_u$  text field, type  $c_s$ .
- 6 In the  $c_\theta$  text field, type  $c_s$ .

#### *Planar Joint: Support 1*

In the **Model Builder** window, click **Planar Joint: Support 1**.

#### *Friction 1*

- 1 In the **Physics** toolbar, click  **Attributes** and choose **Friction**.
- 2 In the **Settings** window for **Friction**, locate the **Friction** section.
- 3 In the  $\mu$  text field, type  $\mu$ .

Create three more planar joints by duplicating **Planar Joint: Support 1**.

#### *Planar Joint: Support 2*

- 1 Right-click **Planar Joint: Support 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Planar Joint**, type Planar Joint: Support 2 in the **Label** text field.

#### *Center of Joint: Boundary 1*

- 1 In the **Model Builder** window, expand the **Planar Joint: Support 2** node, then click **Center of Joint: Boundary 1**.
- 2 In the **Settings** window for **Center of Joint: Boundary**, locate the **Boundary Selection** section.
- 3 Click  **Clear Selection**.
- 4 Select Boundary 52 only.

#### *Planar Joint: Support 3*

- 1 In the **Model Builder** window, right-click **Planar Joint: Support 2** and choose **Duplicate**.

- 2 In the **Settings** window for **Planar Joint**, type Planar Joint: Support 3 in the **Label** text field.

*Center of Joint: Boundary 1*

- 1 In the **Model Builder** window, expand the **Planar Joint: Support 3** node, then click **Center of Joint: Boundary 1**.
- 2 In the **Settings** window for **Center of Joint: Boundary**, locate the **Boundary Selection** section.
- 3 Click  **Clear Selection**.
- 4 Select Boundary 58 only.

*Planar Joint: Support 4*

- 1 In the **Model Builder** window, right-click **Planar Joint: Support 3** and choose **Duplicate**.
- 2 In the **Settings** window for **Planar Joint**, type Planar Joint: Support 4 in the **Label** text field.

*Center of Joint: Boundary 1*

- 1 In the **Model Builder** window, expand the **Planar Joint: Support 4** node, then click **Center of Joint: Boundary 1**.
- 2 In the **Settings** window for **Center of Joint: Boundary**, locate the **Boundary Selection** section.
- 3 Click  **Clear Selection**.
- 4 Select Boundary 24 only.

Define the connection between the washer and the drum using a **Hinge Joint** node.

*Hinge Joint: Washer-drum*

- 1 In the **Physics** toolbar, click  **Global** and choose **Hinge Joint**.
- 2 In the **Settings** window for **Hinge Joint**, type Hinge Joint: Washer-drum in the **Label** text field.
- 3 Locate the **Attachment Selection** section. From the **Source** list, choose **Rigid Material: Washer**.
- 4 From the **Destination** list, choose **Rigid Material: Drum**.
- 5 Locate the **Axis of Joint** section. Specify the  $\mathbf{e}_0$  vector as

|   |   |
|---|---|
| 0 | x |
| 1 | y |
| 0 | z |

#### *Center of Joint: Boundary I*

- 1 In the **Model Builder** window, expand the **Hinge Joint: Washer-drum** node, then click **Center of Joint: Boundary I**.
- 2 Select Boundary 34 only.

Define a **Hinge Joint** between the drum and the slot to correct the angular position of the balancing mass with respect to the drum.

#### *Hinge Joint: Drum-slot*

- 1 In the **Model Builder** window, right-click **Hinge Joint: Washer-drum** and choose **Duplicate**.
- 2 In the **Settings** window for **Hinge Joint**, type Hinge Joint: Drum-slot in the **Label** text field.
- 3 Locate the **Attachment Selection** section. From the **Source** list, choose **Rigid Material: Drum**.
- 4 From the **Destination** list, choose **Rigid Material: Slot**.

Define a **Prismatic Joint** between the slot and the balancing mass to correct the radial position of the balancing mass. In this model the radial correction is however not included and the relative motion at this joint is fully constrained.

#### *Prismatic Joint: Slot-balancing mass*

- 1 In the **Physics** toolbar, click  **Global** and choose **Prismatic Joint**.
- 2 In the **Settings** window for **Prismatic Joint**, type Prismatic Joint: Slot-balancing mass in the **Label** text field.
- 3 Locate the **Attachment Selection** section. From the **Source** list, choose **Rigid Material: Slot**.
- 4 From the **Destination** list, choose **Rigid Material: Balancing mass**.
- 5 Locate the **Axis of Joint** section. From the list, choose **Select a parallel edge**.

#### *Center of Joint: Boundary I*

- 1 In the **Model Builder** window, expand the **Prismatic Joint: Slot-balancing mass** node, then click **Center of Joint: Boundary I**.
- 2 Select Boundary 34 only.

#### *Joint Axis I*

- 1 In the **Model Builder** window, click **Joint Axis I**.
- 2 Select Edge 86 only.

#### *Prismatic Joint: Slot-balancing mass*

In the **Model Builder** window, click **Prismatic Joint: Slot-balancing mass**.

### *Prescribed Motion 1*

In the **Physics** toolbar, click  **Attributes** and choose **Prescribed Motion**.

### **GLOBAL DEFINITIONS**

Add step functions for activation and deactivation of the control mechanism.

#### *Step 1 (step1)*

- 1 In the **Home** toolbar, click  **Functions** and choose **Global>Step**.
- 2 In the **Settings** window for **Step**, type `step_act` in the **Function name** text field.
- 3 Locate the **Parameters** section. In the **Location** text field, type `7[s]`.
- 4 Click to expand the **Smoothing** section. In the **Size of transition zone** text field, type `0.5`.
- 5 Click  **Plot**.

#### *Step 2 (step\_act2)*

- 1 Right-click **Step 1 (step\_act)** and choose **Duplicate**.
- 2 In the **Settings** window for **Step**, type `step_deact` in the **Function name** text field.
- 3 Locate the **Parameters** section. In the **Location** text field, type `5[deg]`.
- 4 Locate the **Smoothing** section. In the **Size of transition zone** text field, type `10[deg]`.
- 5 Click  **Plot**.

### **DEFINITIONS**

#### *Variables 1*

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Variables** section.
- 3 Click  **Load from File**.
- 4 Browse to the model's Application Libraries folder and double-click the file `washing_machine_walk_variables.txt`.

Define the friction force variables on the support boundaries.

#### *Variables 2*

- 1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Boundary**.
- 4 Select Boundary 18 only.

5 Locate the **Variables** section. In the table, enter the following settings:

| Name | Expression  | Unit | Description                 |
|------|-------------|------|-----------------------------|
| Fsx  | mbd.plj1.Fx | N    | Friction force, x component |
| Fsy  | mbd.plj1.Fy | N    | Friction force, y component |

#### Variables 3

- 1 Right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Boundary**.
- 4 Select Boundary 52 only.
- 5 Locate the **Variables** section. In the table, enter the following settings:

| Name | Expression  | Unit | Description                 |
|------|-------------|------|-----------------------------|
| Fsx  | mbd.plj2.Fx | N    | Friction force, x component |
| Fsy  | mbd.plj2.Fy | N    | Friction force, y component |

#### Variables 4

- 1 Right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Boundary**.
- 4 Select Boundary 58 only.
- 5 Locate the **Variables** section. In the table, enter the following settings:

| Name | Expression  | Unit | Description                 |
|------|-------------|------|-----------------------------|
| Fsx  | mbd.plj3.Fx | N    | Friction force, x component |
| Fsy  | mbd.plj3.Fy | N    | Friction force, y component |

#### Variables 5

- 1 Right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Boundary**.
- 4 Select Boundary 24 only.

5 Locate the **Variables** section. In the table, enter the following settings:

| Name | Expression  | Unit | Description                 |
|------|-------------|------|-----------------------------|
| Fsx  | mbd.plj4.Fx | N    | Friction force, x component |
| Fsy  | mbd.plj4.Fy | N    | Friction force, y component |

Specify the drum rotation and angular correction of the balancing mass.

## MULTIBODY DYNAMICS (MBD)

*Hinge Joint: Washer-drum*

In the **Model Builder** window, under **Component 1 (comp1)>Multibody Dynamics (mbd)** click **Hinge Joint: Washer-drum**.

*Prescribed Motion 1*

- 1 In the **Physics** toolbar, click  **Attributes** and choose **Prescribed Motion**.
- 2 In the **Settings** window for **Prescribed Motion**, locate the **Prescribed Rotational Motion** section.
- 3 From the **Prescribed motion through** list, choose **Angular velocity**.
- 4 In the  $\omega_p$  text field, type omega.

*Hinge Joint: Drum-slot*

In the **Model Builder** window, under **Component 1 (comp1)>Multibody Dynamics (mbd)** click **Hinge Joint: Drum-slot**.

*Prescribed Motion 1*

- 1 In the **Physics** toolbar, click  **Attributes** and choose **Prescribed Motion**.
- 2 In the **Settings** window for **Prescribed Motion**, locate the **Prescribed Rotational Motion** section.
- 3 From the **Prescribed motion through** list, choose **Angular velocity**.
- 4 In the  $\omega_p$  text field, type  $i\_c \cdot \omega_{\text{C}}$ .

## STUDY 1

Add a parametric sweep to perform the analysis with and without active balancing.

*Parametric Sweep*

- 1 In the **Study** toolbar, click  **Parametric Sweep**.
- 2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.
- 3 Click  **Add**.

4 In the table, enter the following settings:

| Parameter name             | Parameter value list | Parameter unit |
|----------------------------|----------------------|----------------|
| i_c (Active control state) | 0 1                  |                |

#### *Step 1: Time Dependent*

- 1 In the **Model Builder** window, click **Step 1: Time Dependent**.
- 2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 3 In the **Output times** text field, type range(0,0.01,9).
- 4 In the **Study** toolbar, click  **Compute**.

## **RESULTS**

### *Displacement (mbd)*

Create more datasets for better visualization in the postprocessing.

#### *Study 1/Parametric Solutions 1 (3) (sol2)*

- 1 In the **Model Builder** window, expand the **Results>Datasets** node.
- 2 Right-click **Results>Datasets>Study 1/Parametric Solutions 1 (sol2)** and choose **Duplicate**.

#### *Selection*

- 1 In the **Results** toolbar, click  **Attributes** and choose **Selection**.
- 2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 Select Domain 1 only.

#### *Study 1/Parametric Solutions 1 (4) (sol2)*

In the **Model Builder** window, under **Results>Datasets** right-click **Study 1/Parametric Solutions 1 (2) (sol2)** and choose **Duplicate**.

#### *Selection*

- 1 In the **Results** toolbar, click  **Attributes** and choose **Selection**.
- 2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 Click in the **Graphics** window and then press Ctrl+A to select all domains.
- 5 Select Domains 2–8 only.
- 6 Clear the **Propagate to lower dimensions** check box.

Follow the instructions to plot the washer displacement and friction forces as shown in [Figure 2](#) and [Figure 3](#) respectively.

#### *Washer Displacement*

- 1 In the **Results** toolbar, click  **3D Plot Group**.
- 2 In the **Settings** window for **3D Plot Group**, type Washer Displacement in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 1/ Parametric Solutions 1 (3) (sol2)**.
- 4 From the **Parameter value (i\_c)** list, choose **0**.
- 5 From the **Time (s)** list, choose **8.9**.
- 6 Locate the **Plot Settings** section. From the **Frame** list, choose **Spatial (x, y, z)**.

#### *Volume 1*

- 1 Right-click **Washer Displacement** and choose **Volume**.
- 2 In the **Settings** window for **Volume**, locate the **Coloring and Style** section.
- 3 Click  **Change Color Table**.
- 4 In the **Color Table** dialog box, select **Rainbow>SpectrumLight** in the tree.
- 5 Click **OK**.

#### *Deformation 1*

- 1 Right-click **Volume 1** and choose **Deformation**.
- 2 In the **Settings** window for **Deformation**, locate the **Scale** section.
- 3 Select the **Scale factor** check box. In the associated text field, type 100.
- 4 In the **Washer Displacement** toolbar, click  **Plot**.
- 5 Click the  **Go to Default View** button in the **Graphics** toolbar.
- 6 Click the  **Zoom Extents** button in the **Graphics** toolbar.

#### *Friction Force*

- 1 In the **Model Builder** window, right-click **Washer Displacement** and choose **Duplicate**.
- 2 In the **Settings** window for **3D Plot Group**, type Friction Force in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 1/ Parametric Solutions 1 (4) (sol2)**.
- 4 In the **Model Builder** window, expand the **Friction Force** node.

### *Deformation I*

- 1 In the **Model Builder** window, expand the **Results>Friction Force>Volume I** node, then click **Deformation I**.
- 2 In the **Settings** window for **Deformation**, locate the **Scale** section.
- 3 In the **Scale factor** text field, type 1.
- 4 In the **Friction Force** toolbar, click  **Plot**.

### *Arrow Surface I*

- 1 In the **Model Builder** window, right-click **Friction Force** and choose **Arrow Surface**.
- 2 In the **Settings** window for **Arrow Surface**, locate the **Expression** section.
- 3 In the **X component** text field, type  $F_{sx}$ .
- 4 In the **Y component** text field, type  $F_{sy}$ .
- 5 In the **Z component** text field, type 0.
- 6 Locate the **Coloring and Style** section.
- 7 Select the **Scale factor** check box. In the associated text field, type 0.04.
- 8 In the **Friction Force** toolbar, click  **Plot**.
- 9 Click the  **Go to Default View** button in the **Graphics** toolbar.
- 10 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Follow these instructions to plot the total imbalance in the rotating and the fixed frame as shown in [Figure 4](#) and [Figure 5](#) respectively.

### *Total Imbalance (Local)*

- 1 In the **Home** toolbar, click  **Add Plot Group** and choose **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type Total Imbalance (Local) in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Study 1/ Parametric Solutions 1 (2) (sol2)**.
- 4 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the **Plot Settings** section.
- 6 Select the **y-axis label** check box. In the associated text field, type Total imbalance (N).
- 7 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

### *Global I*

- 1 Right-click **Total Imbalance (Local)** and choose **Global**.

- 2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp I)>Definitions>Variables>Fxl - Total imbalance, xl component - N**.
- 3 Click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp I)>Definitions>Variables>Fzl - Total imbalance, zl component - N**.
- 4 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Cycle**.
- 5 From the **Width** list, choose **2**.
- 6 In the **Total Imbalance (Local)** toolbar, click  **Plot**.
- 7 Click the  **Zoom Extents** button in the **Graphics** toolbar.

#### *Total Imbalance*

- 1 In the **Model Builder** window, right-click **Total Imbalance (Local)** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, type **Total Imbalance** in the **Label** text field.

#### *Global I*

- 1 In the **Model Builder** window, expand the **Total Imbalance** node, then click **Global I**.
- 2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp I)>Definitions>Variables>Fx - Total imbalance, x component - N**.
- 3 Click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp I)>Definitions>Variables>Fz - Total imbalance, z component - N**.
- 4 In the **Total Imbalance** toolbar, click  **Plot**.
- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Follow the instructions below to plot the slip margin for Support 1, Support 3, and the total slip margin as shown in [Figure 6](#) and [Figure 7](#) respectively.

#### *Slip Margin*

- 1 In the **Model Builder** window, right-click **Total Imbalance** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, type **Slip Margin** in the **Label** text field.
- 3 Locate the **Data** section. From the **Parameter selection (i\_c)** list, choose **First**.
- 4 Locate the **Plot Settings** section. In the **y-axis label** text field, type **Slip margin (N)**.
- 5 Locate the **Legend** section. From the **Position** list, choose **Lower left**.

### *Global I*

- 1 In the **Model Builder** window, expand the **Slip Margin** node, then click **Global I**.
- 2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp I)>Definitions>Variables>Msl I - Slip margin, support I - N**.
- 3 Click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp I)>Definitions>Variables>Msl3 - Slip margin, support 3 - N**.
- 4 In the **Slip Margin** toolbar, click  **Plot**.
- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar.

### *Total Slip Margin*

- 1 In the **Model Builder** window, right-click **Slip Margin** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, type Total Slip Margin in the **Label** text field.
- 3 Locate the **Data** section. From the **Parameter selection (i\_c)** list, choose **All**.
- 4 Locate the **Plot Settings** section. Clear the **y-axis label** check box.

### *Global I*

- 1 In the **Model Builder** window, expand the **Total Slip Margin** node, then click **Global I**.
- 2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp I)>Definitions>Variables>Msl\_tot - Total slip margin - N**.
- 3 Click to expand the **Legends** section. Find the **Include** subsection. Clear the **Description** check box.
- 4 In the **Total Slip Margin** toolbar, click  **Plot**.
- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar.

To plot the angular displacement of the washer as shown in [Figure 8](#), follow the instructions below.

### *Washer Rotation*

- 1 In the **Model Builder** window, right-click **Total Slip Margin** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, type Washer Rotation in the **Label** text field.
- 3 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

### *Global I*

- 1 In the **Model Builder** window, expand the **Washer Rotation** node, then click **Global I**.

- In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Multibody Dynamics>Rigid domains>Rigid Material: Washer>Rigid body rotation (spatial frame) - rad>mbd.rd1.thz - Rigid body rotation, z component**.  
Change the units to degrees from radians.
- Locate the **y-Axis Data** section. In the table, enter the following settings:

| Expression  | Unit | Description                      |
|-------------|------|----------------------------------|
| mbd.rd1.thz | deg  | Rigid body rotation, z component |

- In the **Washer Rotation** toolbar, click  **Plot**.  
Use the following instructions to plot the RPM of the drum and the correction motor as shown in [Figure 9](#).
- Click the  **Zoom Extents** button in the **Graphics** toolbar.

#### Angular Velocity

- In the **Model Builder** window, right-click **Washer Rotation** and choose **Duplicate**.
- In the **Settings** window for **ID Plot Group**, type Angular Velocity in the **Label** text field.
- Locate the **Data** section. From the **Parameter selection (i\_c)** list, choose **Last**.
- Locate the **Plot Settings** section.
- Select the **y-axis label** check box. In the associated text field, type Angular velocity (rpm).

#### Global 1

- In the **Model Builder** window, expand the **Angular Velocity** node, then click **Global 1**.
- In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- In the table, enter the following settings:

| Expression | Unit | Description                       |
|------------|------|-----------------------------------|
| rpm        | 1    | Drum angular velocity (rpm)       |
| rpmC       | 1    | Correction angular velocity (rpm) |

- Locate the **Legends** section. Find the **Include** subsection. Select the **Description** check box.
- In the **Angular Velocity** toolbar, click  **Plot**.
- Click the  **Zoom Extents** button in the **Graphics** toolbar.

Use the following instructions to plot the correction angle shown in [Figure 10](#).

#### *Correction Angle*

- 1 In the **Model Builder** window, right-click **Angular Velocity** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, type **Correction Angle** in the **Label** text field.
- 3 Locate the **Plot Settings** section. Clear the **y-axis label** check box.

#### *Global I*

- 1 In the **Model Builder** window, expand the **Correction Angle** node, then click **Global I**.
- 2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp I)>Definitions>Variables>d\_th - Correction angle - rad**.
- 3 Locate the **y-Axis Data** section. In the table, enter the following settings:

| Expression | Unit | Description      |
|------------|------|------------------|
| d_th       | deg  | Correction angle |

- 4 Locate the **Legends** section. Clear the **Show legends** check box.
- 5 In the **Correction Angle** toolbar, click  **Plot**.
- 6 Click the  **Zoom Extents** button in the **Graphics** toolbar.

