

# Out-of-Plane Heat Transfer for a Thin Plate

# Introduction

Modeling heat transfer in a thin rectangular metal plate, this example demonstrates how to use the **Out-of-Plane Heat Flux** and **Out-of-Plane Radiation** features in the Heat Transfer interfaces. Because the plate thickness is only 1/100th of its length and width, it is appropriate to simulate the process using a 2D approximation where the temperature is assumed to be constant along the thickness. The plate has a fixed temperature at one end and is isolated at the other end. A surrounding liquid cools the plate by convection. In addition, the model considers surface-to-ambient radiation. To check the validity of the 2D approximation, the example finishes by setting up a full 3D model and comparing the results of the two versions.

# Model Definition

The plate has a square shape of side length 1 m while its thickness is only 10 mm. Figure 1 illustrates the simplification made from 3D to 2D.



Figure 1: Model geometry reduction from 3D to 2D.

The stationary heat transfer equation for the 2D problem is

$$\begin{aligned} \nabla \cdot (-d_z k \nabla T) &= h_{\mathrm{u}} (T_{\mathrm{ext, u}}^4 - T^4) + h_{\mathrm{d}} (T_{\mathrm{ext, d}}^4 - T^4) \\ &+ \varepsilon_{\mathrm{u}} \sigma (T_{\mathrm{amb, u}}^4 - T^4) + \varepsilon_{\mathrm{d}} \sigma (T_{\mathrm{amb, d}}^4 - T^4) \end{aligned}$$

In this case the problem is symmetric, that is, the upside constants are equal to the downside ones. The heat transfer film coefficient,  $h=h_u=h_d$ , is equal to  $10 \text{ W/(m}^2 \cdot \text{K})$  and the emissivity,  $\varepsilon = \varepsilon_u = \varepsilon_d$ , is equal to 0.5. The ambient and external temperatures,  $T_{\text{amb}} = T_{\text{amb},u} = T_{\text{amb},d}$  and  $T_{\text{ext}} = T_{\text{ext},u} = T_{\text{ext},d}$ , are both set to 300 K. The model uses the default material properties for copper from the built-in material library. The variable  $d_z$  denotes the plate thickness, and  $\sigma$  is the Stefan-Boltzmann constant.

Finally, set the left boundary to a fixed temperature of 800 K and assume that the other boundaries are thermally insulated.

# Results and Discussion



Figure 2 shows the steady-state temperature distribution.

Figure 2: Stationary temperature distribution in the plate.

Figure 3 shows the temperature profile in the full 3D case.



Figure 3: Temperature profile in the full 3D case.



Figure 4: Comparison of the temperature (from left to right in the plate) from both the 2D out-of-plane model (blue) and the full 3D model (green stars). The curves coincide to a very high degree, making it difficult to tell them apart.

It is interesting to compare the results from the 2D approximation with the equivalent 3D model. Figure 4 shows the temperature along the bottom edge of the 2D model and the corresponding 3D result.

**Application Library path:** Heat\_Transfer\_Module/Verification\_Examples/ thin\_plate

# Modeling Instructions

From the File menu, choose New.

# NEW

In the New window, click 🙆 Model Wizard.

# MODEL WIZARD

- I In the Model Wizard window, click 9 2D.
- 2 In the Select Physics tree, select Heat Transfer>Heat Transfer in Solids (ht).
- 3 Click Add.
- 4 Click 🔿 Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click **M** Done.

# **GLOBAL DEFINITIONS**

# Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click 📂 Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file thin\_plate\_parameters.txt.

# GEOMETRY I

Square 1 (sq1)

- I In the **Geometry** toolbar, click **Square**.
- 2 In the Settings window for Square, locate the Size section.

- 3 In the Side length text field, type length.
- 4 Click 🟢 Build All Objects.
- **5** Click the **Graphics** toolbar.

#### ADD MATERIAL

- I In the Home toolbar, click 🙀 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Copper.
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

#### HEAT TRANSFER IN SOLIDS (HT)

- I In the Model Builder window, under Component I (comp1) click Heat Transfer in Solids (ht).
- 2 In the Settings window for Heat Transfer in Solids, locate the Physical Model section.
- **3** In the  $d_z$  text field, type thickness.

#### Out-of-Plane Heat Flux I

- I In the Physics toolbar, click 🔵 Domains and choose Out-of-Plane Heat Flux.
- **2** Select Domain 1 only.
- **3** In the **Settings** window for **Out-of-Plane Heat Flux**, locate the **Upside Inward Heat Flux** section.
- 4 From the Flux type list, choose Convective heat flux.
- **5** In the  $h_{\rm u}$  text field, type h\_c.
- **6** In the  $T_{\text{ext,u}}$  text field, type T\_ext.
- 7 Locate the Downside Inward Heat Flux section. From the Flux type list, choose Convective heat flux.
- **8** In the  $h_d$  text field, type h\_c.
- **9** In the  $T_{\text{ext.d}}$  text field, type T\_ext.

## Out-of-Plane Radiation 1

- I In the Physics toolbar, click 🔵 Domains and choose Out-of-Plane Radiation.
- **2** Select Domain 1 only.
- 3 In the Settings window for Out-of-Plane Radiation, locate the Upside Parameters section.
- **4** From the  $\varepsilon_{\rm u}$  list, choose **User defined**. In the associated text field, type **0.5**.

- **5** In the  $T_{\text{amb.u}}$  text field, type T\_ext.
- 6 Locate the Downside Parameters section. From the  $\varepsilon_d$  list, choose User defined. In the associated text field, type 0.5.
- 7 In the  $T_{\text{amb.d}}$  text field, type T\_ext.

### Temperature I

- I In the Physics toolbar, click Boundaries and choose Temperature.
- 2 Select Boundary 1 only.
- 3 In the Settings window for Temperature, locate the Temperature section.
- **4** In the  $T_0$  text field, type T\_hot.

# STUDY I

In the **Home** toolbar, click **= Compute**.

# RESULTS

## Temperature (ht)

The first default plot shows the temperature distribution (Figure 2).

I Click the **Click the** I **Zoom Extents** button in the **Graphics** toolbar.

Now add a second model for the full 3D problem.

# ADD COMPONENT

In the Model Builder window, right-click the root node and choose Add Component>3D.

#### ADD PHYSICS

- I In the Home toolbar, click 🙀 Add Physics to open the Add Physics window.
- 2 Go to the Add Physics window.
- 3 In the tree, select Heat Transfer>Heat Transfer in Solids (ht).
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check box for Study 1.
- 5 Click Add to Component 2 in the window toolbar.
- 6 In the Home toolbar, click 🖄 Add Physics to close the Add Physics window.

# ADD STUDY

- I In the Home toolbar, click  $\stackrel{\text{res}}{\longrightarrow}$  Add Study to open the Add Study window.
- 2 Go to the Add Study window.

- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check box for Heat Transfer in Solids (ht).
- 5 Click Add Study in the window toolbar.
- 6 In the Model Builder window, click the root node.
- 7 In the Home toolbar, click  $\sim 1$  Add Study to close the Add Study window.

#### **GEOMETRY 2**

In the Model Builder window, under Component 2 (comp2) click Geometry 2.

Block I (blkI)

- I In the **Geometry** toolbar, click 🗍 **Block**.
- 2 In the Settings window for Block, locate the Size and Shape section.
- 3 In the Width text field, type length.
- 4 In the **Depth** text field, type length.
- 5 In the Height text field, type thickness.

## ADD MATERIAL

- I In the Home toolbar, click 🙀 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Copper.
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

#### HEAT TRANSFER IN SOLIDS 2 (HT2)

In the Model Builder window, under Component 2 (comp2) click Heat Transfer in Solids 2 (ht2).

### Heat Flux 1

- I In the Physics toolbar, click 🔚 Boundaries and choose Heat Flux.
- 2 Select Boundaries 3 and 4 only.
- 3 In the Settings window for Heat Flux, locate the Heat Flux section.
- 4 From the Flux type list, choose Convective heat flux.
- **5** In the *h* text field, type h\_c.
- **6** In the  $T_{\text{ext}}$  text field, type T\_ext.

#### Surface-to-Ambient Radiation I

- I In the Physics toolbar, click 🔚 Boundaries and choose Surface-to-Ambient Radiation.
- **2** Select Boundaries 3 and 4 only.
- **3** In the Settings window for Surface-to-Ambient Radiation, locate the Surface-to-Ambient Radiation section.
- **4** From the  $\varepsilon$  list, choose **User defined**. In the associated text field, type **0.5**.
- **5** In the  $T_{\text{amb}}$  text field, type T\_ext.

#### Temperature 1

- I In the Physics toolbar, click 🔚 Boundaries and choose Temperature.
- 2 Select Boundary 1 only.
- 3 In the Settings window for Temperature, locate the Temperature section.
- **4** In the  $T_0$  text field, type T\_hot.

# STUDY 2

In the **Home** toolbar, click  $\equiv$  **Compute**.

# RESULTS

Temperature (ht2)

The default plots show the temperature distribution of the full 3D problem (compare with Figure 3) and the isothermal contours.

To compare the temperature profiles along the bottom edge for the 2D approximation and 3D model, as done in Figure 4, follow the steps given below.

### Models Comparison

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Models Comparison in the Label text field.

Line Graph 1

- I Right-click Models Comparison and choose Line Graph.
- 2 Select Boundary 2 only.
- 3 In the Settings window for Line Graph, click Replace Expression in the upper-right corner of the x-Axis Data section. From the menu, choose Component I (compl)>Geometry> Coordinate>x x-coordinate.
- 4 Click to expand the Legends section. Select the Show legends check box.

- 5 From the Legends list, choose Manual.
- 6 In the table, enter the following settings:

#### Legends

2D Model

7 In the Models Comparison toolbar, click 💽 Plot.

Models Comparison

In the Model Builder window, click Models Comparison.

Line Graph 2

- I In the Models Comparison toolbar, click 📐 Line Graph.
- 2 In the Settings window for Line Graph, locate the Data section.
- 3 From the Dataset list, choose Study 2/Solution 2 (3) (sol2).
- 4 Select Edge 3 only.
- 5 Click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component 2 (comp2)>Heat Transfer in Solids 2>Temperature>T2 Temperature K.
- 6 Click Replace Expression in the upper-right corner of the x-Axis Data section. From the menu, choose Component 2 (comp2)>Geometry>Coordinate>x x-coordinate.
- 7 Click to expand the Title section. From the Title type list, choose None.
- 8 Click to expand the Coloring and Style section. Find the Line style subsection. From the Line list, choose None.
- 9 Find the Line markers subsection. From the Marker list, choose Asterisk.
- **IO** From the **Positioning** list, choose **Interpolated**.
- II Locate the Legends section. Select the Show legends check box.
- 12 From the Legends list, choose Manual.
- **I3** In the table, enter the following settings:

#### Legends

3D Model

**I4** In the **Models Comparison** toolbar, click **ID Plot**.