Created in COMSOL Multiphysics 6.1

Nonisothermal Laminar Flow in a Circular Tube

Introduction

This validation model of laminar airflow through a tube validates the heat transfer coefficient obtained from the simulation against Nusselt-number-based correlation functions. The simulation results are in good agreement with experimental measurements.

Model Definition

The tube is modeled as a 2D axisymmetric geometry. The tube has a diameter of 0.05 m and a length of 3 m. A coupled heat transfer and fluid flow problem is solved using the Nonisothermal Flow interface.

At the inlet, a laminar velocity profile U with an average velocity U_{av} of 0.1 m/s is applied using the normal inflow option:

$$U = 1.5U_{\mathrm{av}}\left(1-4\left(\frac{r}{b}\right)^2\right)$$

where r denotes the radial distance from the tube center and b the tube's diameter. The expression gives the typical parabolic velocity profile for fully developed laminar flow. The air enters with a temperature T_0 of 283 K.

At the cylinder wall, a constant heat flux q_{w} of 10 W/m² is applied.

NUSSELT NUMBER CORRELATIONS

Two different Nusselt number correlations are used to validate the numerical results.

First, in regions with fully developed laminar flow with a radial temperature profile, a constant Nusselt number $\mathrm{Nu_c}$ can be defined as follows:

$$\mathrm{Nu_c} = \frac{hD_{\mathrm{h}}}{k}$$

where k (SI unit: W/(m·K)) denotes the thermal conductivity, D_{h} (SI unit: m) the hydraulic diameter, and h (SI unit: W/(m^2·K)) the heat transfer coefficient. In the case of a tube with uniform surface heat flux, $\mathrm{Nu} = 4.36$ (Ref. 1, p. 507).

Alternatively, a local Nusselt number $\mathrm{Nu_l}$ can be defined, based on the z-position along the cylinder, to describe both the entrance and fully developed regions of the flow (Ref. 2, p. 304):

$$\frac{\mathrm{Nu_l}}{4.364\left[1+\left(\frac{\mathrm{Gz}}{29.6}\right)^2\right]^{1/6}}$$

$$= \left[1+\left(\frac{\mathrm{Gz}/19.04}{\left[1+\left(\frac{\mathrm{Pr}}{0.0207}\right)^{2/3}\right]^{1/2}\left[1+\left(\frac{\mathrm{Gz}}{29.6}\right)^2\right]^{1/3}}\right)^{3/2}\right]^{1/3}$$

where Pr is the Prandt number, and the Graetz number Gz is defined by:

$$\mathrm{Gz} = \frac{\pi}{4}\cdot\frac{\mathrm{Re_b}\cdot\mathrm{Pr}\cdot b}{z}$$

with $\mathrm{Re_b}$ the Reynolds number associated to the tube diameter b.

Results and Discussion

The velocity field is shown in Figure 1 and the temperature field in Figure 2. Both are plotted in a scaled view to get a clearer visualization of the results.

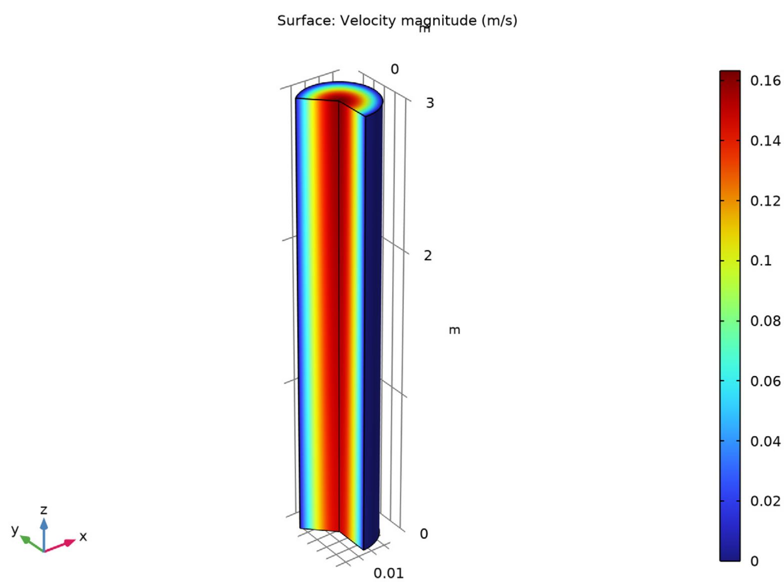

Figure 1: Velocity field.

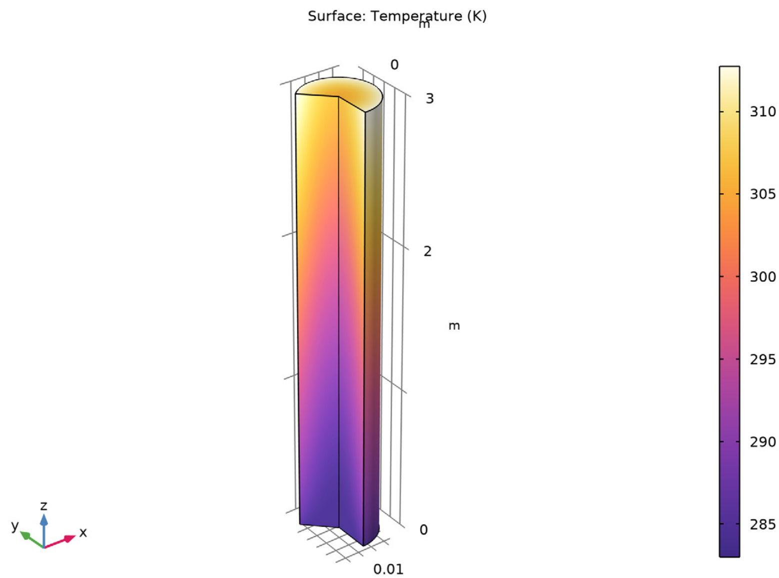

Figure 2: Temperature field.

The comparison of the computed heat transfer coefficient with the Nusselt number correlations shows that the Local Nusselt number provides a good approximation over the whole cylinder. On the other hand, constant Nusselt number represents the region where velocity and temperature profile are fully developed (Figure 3).

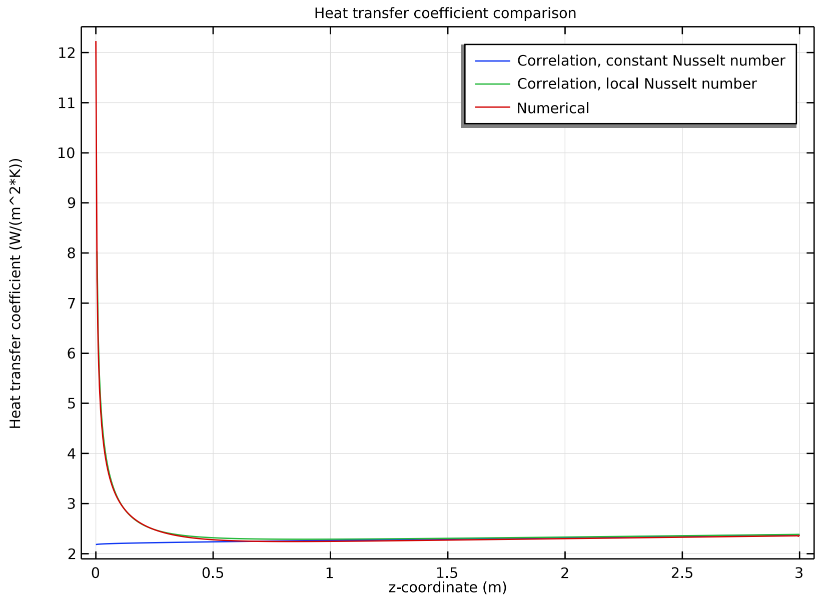

Figure 3: Comparison of the computed heat transfer coefficient with the heat transfer coefficient estimation based on Nusselt number correlations.

References

1. F.P. Incropera, D.P. DeWitt, T.L. Bergman, and A.S. Lavine, *Fundamentals of Heat and Mass Transfer*, 6th Edition, John Wiley & Sons, 2006.

2. A. Bejan et al., *Heat Transfer Handbook*, John Wiley & Sons, 2003.

Application Library path: `Heat_Transfer_Module/Verification_Examples/circular_tube_nitf_laminar`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **Fluid Flow>Nonisothermal Flow>Laminar Flow**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
L	3[m]	3 m	Length
b	0.05[m]	0.05 m	Height
T0	283[K]	283 K	Inlet temperature
U_av	0.1[m/s]	0.1 m/s	Average inlet velocity
qw	10[W/m^2]	10 W/m²	Wall heat flux
Tw	293[K]	293 K	Wall temperature

DEFINITIONS

Variables 1

Define several variables: A variable for the inlet velocity profile, a variable for the bulk temperature which is a radial weighted temperature, and similarly a variable for the bulk velocity. Finally, add variables to compare the simulation results with the literature values.

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
U	1.5*U_av*(1-4*(r/b)^2)	m/s	Inlet velocity
Tb	integrate(comp1.at2(r,z,2*pi*r*w*T),r,0,b/2)/integrate(comp1.at2(r,z,2*pi*r*w),r,0,b/2)	K	Bulk temperature
Ub	integrate(comp1.at2(r,z,2*pi*r*w),r,0,b/2)/(pi*(b/2)^2)	m/s	Bulk velocity
Tc	comp1.at2(0,z,T)	K	Center line temperature
Pr	ht.Cp*spf.mu/ht.kmean		Prandtl number
Re_D	nitf1.rho*Ub*b/spf.mu		Reynolds number
Gz	b*Re_D*Pr/z*pi/4		Graetz number
Nu_D	(1+(Gz/19.04/((1+(Pr/0.0207)^2/3)^1/2*(1+(Gz/29.6)^2)^1/3))^(3/2))^(1/3)*4.364*(1+(Gz/29.6)^2)^(1/6)		Local Nusselt number

GEOMETRY 1

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type b/2.

4 In the **Height** text field, type L.

5 Click **Build All Objects**.

ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

5 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

LAMINAR FLOW (SPF)

Inlet 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Laminar Flow (spf)** and choose **Inlet**.

2 Select Boundary 2 only.

3 In the **Settings** window for **Inlet**, locate the **Velocity** section.

4 In the U_0 text field, type `U`.

Outlet 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Outlet**.

2 Select Boundary 3 only.

HEAT TRANSFER IN FLUIDS (HT)

In the **Model Builder** window, under **Component 1 (comp1)** click **Heat Transfer in Fluids (ht)**.

Inflow 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Inflow**.

2 Select Boundary 2 only.

3 In the **Settings** window for **Inflow**, locate the **Upstream Properties** section.

4 In the T_{ustr} text field, type `T0`.

Outflow 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Outflow**.

2 Select Boundary 3 only.

Heat Flux 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Heat Flux**.

2 Select Boundary 4 only.

3 In the **Settings** window for **Heat Flux**, locate the **Heat Flux** section.

4 In the q_0 text field, type `qw`.

MESH 1

Mapped 1

In the **Mesh** toolbar, click **Mapped**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 Select Boundary 4 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type 600.

Distribution 2

1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.

2 Click the **Zoom Extents** button in the **Graphics** toolbar.

3 Select Boundaries 2 and 3 only.

4 In the **Settings** window for **Distribution**, locate the **Distribution** section.

5 From the **Distribution type** list, choose **Predefined**.

6 In the **Number of elements** text field, type 33.

7 In the **Element ratio** text field, type 5.

8 Click **Build All**.

STUDY 1

In the **Home** toolbar, click **Compute**.

RESULTS

Velocity, 3D (spf)

For better visualization of the results, use a scaled the view.

View 3D 2

In the **Model Builder** window, expand the **Results>Views** node.

Camera

1 In the **Model Builder** window, expand the **View 3D 2** node, then click **Camera**.

2 In the **Settings** window for **Camera**, locate the **Camera** section.

3 From the **View scale** list, choose **Manual**.

4 In the **z scale** text field, type 0.1.

5 Click **Update**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar.

Velocity, 3D (spf)

Click the **Zoom Extents** button in the **Graphics** toolbar.

Heat Transfer Coefficient

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type Heat Transfer Coefficient in the **Label** text field.
3 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.
4 In the **Title** text area, type Heat transfer coefficient comparison.
5 Locate the **Plot Settings** section.
6 Select the **y-axis label** check box. In the associated text field, type Heat transfer coefficient (W/(m^2*K)).

Line Graph 1

1 Right-click **Heat Transfer Coefficient** and choose **Line Graph**.
2 Select Boundary 4 only.
3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
4 In the **Expression** text field, type 4.36*ht.krr/b.
5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
6 In the **Expression** text field, type z.
7 Click to expand the **Legends** section. Select the **Show legends** check box.
8 From the **Legends** list, choose **Manual**.
9 In the table, enter the following settings:

Legends
Correlation, constant Nusselt number

Line Graph 2

1 In the **Model Builder** window, right-click **Heat Transfer Coefficient** and choose **Line Graph**.
2 Select Boundary 4 only.
3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.
4 In the **Expression** text field, type Nu_D*ht.kmean/b.
5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
6 In the **Expression** text field, type z.
7 Locate the **Legends** section. Select the **Show legends** check box.
8 From the **Legends** list, choose **Manual**.
9 In the table, enter the following settings:

Legends
Correlation, local Nusselt number

Line Graph 3

1 Right-click **Heat Transfer Coefficient** and choose **Line Graph**.

2 Select Boundary 4 only.

3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `qw/(T-Tb)`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `z`.

7 Locate the **Legends** section. Select the **Show legends** check box.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
`Numerical`

Heat Transfer Coefficient

1 In the **Model Builder** window, click **Heat Transfer Coefficient**.

2 In the **Heat Transfer Coefficient** toolbar, click **Plot**.