

# Semibatch Polymerization

This model is licensed under the COMSOL Software License Agreement 6.1. All trademarks are the property of their respective owners. See www.comsol.com/trademarks.

# Introduction

As reactant monomer converts into polymer chains, the density of the reacting mixture often changes notably. This example looks at how this effect impacts the total production of polymer in a process. The liquid phase polymerization takes place in a semibatch reactor, where two operating conditions are compared. In the first scenario, the feed of monomer to the reactor is turned off once the maximum volume capacity is reached. In a second scenario, the feed of monomer is allowed to continuously compensate for the volume change due to chemical reaction.

The model illustrates the use of the Semibatch reactor type, which is predefined in the Reaction Engineering interface in the Chemical Reaction Engineering Module. It also shows how to set timed events, in this case for controlling the reactant feed to the reactor. This example reproduces results found in Ref. 1.

# Model Definition

A liquid phase polymerization can be modeled as a first order irreversible reaction:

$$M \xrightarrow{k_l} P$$
$$r_1 = k_1 c_M$$

In the above equations, M denotes the monomer, P the polymer, r is the reaction rate (SI unit: mol/(m<sup>3</sup>·s)), k is the rate constant (SI unit: 1/s), and  $c_M$  is the concentration of the monomer. This process takes place in the presence of water.

The polymerization takes place in a semibatch reactor with a volume capacity of 20 m<sup>3</sup>. Initially the reactor is charged with 10 m<sup>3</sup> of water. Pure monomer enters the reactor with a volumetric flow rate of  $v_f = 1$  m<sup>3</sup>/min.

Figure 1 shows a schematic representation of the semibatch reactor.



Figure 1: The Semibatch reactor is a predefined reactor type in the Reaction Engineering interface.

The following mass balance describes the semibatch reactor:

$$\frac{d(c_i V_r)}{dt} = v_{f,i} c_{f,i} + R_i V_r$$
(1)

In Equation 1,  $c_i$  is species molar concentration (SI unit: mol/m<sup>3</sup>),  $c_{f,i}$  is the species concentration (SI unit: mol/m<sup>3</sup>) of the associated feed stream  $v_{f,i}$  (SI unit: m<sup>3</sup>/s), and  $R_i$  denotes the species rate expression (SI unit: mol/(m<sup>3</sup>·s)).  $V_r$  labels the reactor volume (SI unit: m<sup>3</sup>) and is a function of time. For ideal mixtures:

$$\frac{dV_r}{dt} = \sum v_{f,i} + v_p$$

where  $v_p$  is the volumetric production rate due to chemical reaction:

$$v_p = \sum_j \sum_i v_{ij} \frac{M_i}{\rho_i} r_j V_r$$
<sup>(2)</sup>

In Equation 2,  $v_{ij}$  is the stoichiometric coefficient of species *i* in reaction *j*,  $M_i$  denotes the species molecular weight (SI unit: kg/mol),  $\rho_i$  is the species density (SI unit: kg/m<sup>3</sup>), and  $r_j$  is the reaction rate (SI unit: mol/(m<sup>3</sup>·s)) of reaction *j*.

In the present example, the density of the monomer is  $800 \text{ kg/m}^3$ ,  $1100 \text{ kg/m}^3$  for the polymer, and  $1000 \text{ kg/m}^3$  for water. Hence, as polymer is being formed, the volume of

the reacting mixture decreases ( $v_p$  is negative). The model investigates two operating conditions:

- Operating condition 1 The monomer feed  $(1 \text{ m}^3/\text{min})$  is shut off once the reactor volume reaches 20 m<sup>3</sup>, which occurs after 11.2 minutes. The reaction is then allowed to go to completion.
- Operating condition 2— The monomer feed is adjusted to keep the reactor filled while the reaction goes to completion. This is accomplished by setting the volumetric feed equal to  $-v_p$ , for t > 11.2 minutes.

Results

Figure 2 illustrates the volumetric flow rate of the feed stream,  $v_{f}$ .



Figure 2: The volumetric flow rate of the feed stream  $(m^3/s)$  as function of time (minutes) for operating condition 1 (solid line) and 2 (dash-dotted line).

Figure 3 shows the reactor volume as function of the runtime, illustrating the two operating conditions listed above.



Figure 3: The reactor volume  $(m^3)$  as function of time (minutes) for operating condition 1 (solid line) and 2 (dash-dotted line).

Figure 4 shows the total mass of monomer in the reactor,  $m_M$  (kg), as evaluated by the expression:

$$m_M = c_M V_r M_M$$



Figure 4: The total monomer mass in the reactor volume (kg) as function of time (minutes) for operating condition 1 (solid line) and 2 (dash-dotted line).

It is straightforward to compare the amount of produced polymer as a result of the different operating conditions. For both cases the reaction has run to completion after approximately 50 minutes. At this time, the total volume of the reacting mixture is  $18.2 \text{ m}^3$  for operating condition 1 and 20 m<sup>3</sup> for operating condition 2.

The relative increase in polymer production using operating condition 2 compared to condition 1 is then:

$$\frac{m_{P,2} - m_{P,1}}{m_{P,1}} = \frac{c_{P,2}V_{r,2}M_P - c_{P,1}V_{r,1}M_P}{c_{P,1}V_{r,1}M_P} = \frac{10.5 - 8.8}{8.8} = 19.3\%$$

# Reference

1. J.B. Rawlings and J.G. Ekerdt, *Chemical Reactor Analysis and Design Fundamentals*, Nob Hill Publishing, example 4.3, pp. 139–144, 2004.

**Application Library path:** Chemical\_Reaction\_Engineering\_Module/ Ideal\_Tank\_Reactors/semibatch\_polymerization

# Modeling Instructions

From the File menu, choose New.

#### NEW

In the New window, click 🔗 Model Wizard.

# MODEL WIZARD

- I In the Model Wizard window, click 0D.
- 2 In the Select Physics tree, select Chemical Species Transport>Reaction Engineering (re).
- 3 Click Add.
- 4 Click 🔿 Study.
- 5 In the Select Study tree, select General Studies>Time Dependent.
- 6 Click **M** Done.

#### **GLOBAL DEFINITIONS**

Add a set of model parameters by importing their definitions from a text file provided with the **Model Library**.

#### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click 📂 Load from File.
- 4 Browse to the model's Application Libraries folder and double-click the file semibatch\_polymerization\_parameters.txt.

# Step I (step I)

Add a step function that regulates the volumetric feed rate during the operations.

- I In the Home toolbar, click f(x) Functions and choose Global>Step.
- 2 In the Settings window for Step, locate the Parameters section.
- **3** In the **Location** text field, type t\_cond.

#### REACTION ENGINEERING (RE)

- I In the Model Builder window, under Component I (compl) click Reaction Engineering (re).
- 2 In the Settings window for Reaction Engineering, locate the Reactor section.
- 3 From the Reactor type list, choose Semibatch.
- 4 Click to expand the Mixture Properties section. From the Phase list, choose Liquid.

#### Reaction 1

- I In the **Reaction Engineering** toolbar, click 👗 **Reaction**.
- 2 In the Settings window for Reaction, locate the Reaction Formula section.
- **3** In the **Formula** text field, type M=>P.
- **4** Locate the **Rate Constants** section. In the  $k^{f}$  text field, type kf\_reaction.

#### Species: M

- I In the Model Builder window, click Species: M.
- 2 In the Settings window for Species, locate the Chemical Formula section.
- **3** In the *M* text field, type Mm\_M.
- **4** In the  $\rho$  text field, type density\_M.

#### Species: P

- I In the Model Builder window, click Species: P.
- 2 In the Settings window for Species, locate the Chemical Formula section.
- **3** In the M text field, type Mm\_P.
- **4** In the  $\rho$  text field, type density\_P.

The reaction takes place in water.

## Species 1

- I In the Reaction Engineering toolbar, click 📩 Species.
- 2 In the Settings window for Species, locate the Name section.
- **3** In the text field, type H20.
- **4** Locate the **Chemical Formula** section. In the M text field, type Mm\_H20.
- **5** In the  $\rho$  text field, type density\_H20.

## DEFINITIONS

Add the two filling conditions as variables dependent on the step function and the monomer mass expression. This is retrieved from a data text file provided with the **Model Library**.

# Variables I

- I In the Model Builder window, under Component I (compl) right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- 3 Click **b** Load from File.
- 4 Browse to the model's Application Libraries folder and double-click the file semibatch\_polymerization\_variables.txt.

#### **REACTION ENGINEERING (RE)**

Feed Inlet I

- I In the Reaction Engineering toolbar, click 🚽 Feed Inlet.
- 2 In the Settings window for Feed Inlet, locate the Feed Inlet Properties section.
- **3** In the  $v_{f}$  text field, type vfs.
- **4** Locate the **Feed Inlet Concentration** section. In the **Feed inlet concentration** table, enter the following settings:

Species	Concentration (mol/m <sup>3</sup> )	
H2O	cinlet_H2O	
Μ	cinlet_M	

Initial Values 1

- I In the Model Builder window, click Initial Values I.
- 2 In the Settings window for Initial Values, locate the General Parameters section.
- **3** In the  $V_{r0}$  text field, type Vr\_init.
- **4** Locate the **Volumetric Species Initial Values** section. In the table, enter the following settings:

Species	Concentration (mol/m^3)	
H2O	cinit_H2O	

## STUDY I

- Step 1: Time Dependent
- I In the Model Builder window, under Study I click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Study Settings section.
- **3** In the **Output times** text field, type range(0,0.1,3000).

**4** In the **Home** toolbar, click **= Compute**.

# RESULTS

Global I

- I In the Model Builder window, expand the Concentration (re) node, then click Global I.
- 2 In the Settings window for Global, locate the x-Axis Data section.
- 3 From the Parameter list, choose Expression.
- **4** In the **Expression** text field, type t.
- 5 From the Unit list, choose min.
- 6 In the Concentration (re) toolbar, click 🗿 Plot.

# STUDY I

Use the **Parametric Sweep** feature to investigate the difference when the compensating fill is turned on.

Parametric Sweep

- I In the Study toolbar, click **Parametric Sweep**.
- 2 In the Settings window for Parametric Sweep, locate the Study Settings section.
- 3 Click + Add.
- **4** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
fill (Control parameter for filling)	0 1	

**5** In the **Study** toolbar, click **= Compute**.

# RESULTS

Volumetric feed rate

In the **Settings** window for **ID Plot Group**, type Volumetric feed rate in the **Label** text field.

Global I

- I In the Model Builder window, expand the Volumetric feed rate node, then click Global I.
- In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
   Reaction Engineering>re.sumvf Sum of volume flows m<sup>3</sup>/s.
- 3 Click to expand the Title section. From the Title type list, choose Manual.

- 4 In the Title text area, type Global: Volumetric feed rate (m<sup>3</sup>/s).
- 5 Locate the x-Axis Data section. From the Axis source data list, choose Time.
- 6 From the Parameter list, choose Expression.
- 7 In the **Expression** text field, type t.
- 8 From the Unit list, choose min.
- 9 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Cycle**.
- **IO** From the **Color** list, choose **Blue**.
- II From the Width list, choose 2.
- 12 Click to expand the Legends section. From the Legends list, choose Manual.
- **I3** In the table, enter the following settings:

#### Legends

Operating condition 1

Operating condition 2

**I4** In the **Volumetric feed rate** toolbar, click **I Plot**.

#### Reactor volume

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Reactor volume in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study I/ Parametric Solutions I (sol2).
- 4 Locate the Legend section. From the Position list, choose Lower right.

#### Global I

- I Right-click Reactor volume and choose Global.
- In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
   Reaction Engineering>re.Vr Reactor volume m<sup>3</sup>.
- 3 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **4** In the **Expression** text field, type **t**.
- 5 From the Unit list, choose min.
- 6 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose Cycle.
- 7 From the Color list, choose Blue.

- 8 From the Width list, choose 2.
- 9 Locate the Legends section. From the Legends list, choose Manual.
- **IO** In the table, enter the following settings:

#### Legends

Operating condition 1

Operating condition 2

II In the **Reactor volume** toolbar, click **I** Plot.

Monomer mass

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Monomer mass in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study I/ Parametric Solutions I (sol2).

#### Global I

- I Right-click Monomer mass and choose Global.
- 2 In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>Definitions> Variables>m\_mon - Monomer mass - kg.
- 3 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **4** In the **Expression** text field, type t.
- 5 From the Unit list, choose min.
- 6 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose Cycle.
- 7 From the Color list, choose Blue.
- 8 From the Width list, choose 2.
- 9 Locate the Legends section. From the Legends list, choose Manual.
- **IO** In the table, enter the following settings:

#### Legends

Operating condition 1

Operating condition 2

II In the Monomer mass toolbar, click 💿 Plot.