

Eigenmodes in a Muffler

Introduction

In this example, you compute the propagating modes in the chamber of an automotive muffler. The geometry is a cross section of the chamber in the Absorptive Muffler example.

The purpose of the model is to study the shape of the propagating modes and to find their cutoff frequencies. As discussed in the documentation of the Absorptive Muffler example, some of the modes significantly affect the damping of the muffler at frequencies above their cutoff. In this model, you study modes with cutoff frequencies up to 1500 Hz.

Model Definition

The muffler chamber has a race track shaped cross section, as seen in Figure 1. In this model, the chamber is considered to be hollow and field with air at atmospheric pressure.



Figure 1: The model geometry.

The wave numbers and mode shapes through a cross section of the chamber are found as the solution of an eigenvalue problem for the acoustic pressure *p*:

$$\nabla \cdot \left(-\frac{\nabla p(x,y)}{\rho_0}\right) - \left(\frac{\omega^2}{\rho_0 c^2} - \frac{\kappa_z^2}{\rho_0}\right) p(x,y) = 0$$

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where ρ_0 is the density, *c* the speed of sound, κ_z the out-of-plane wave number, and $\omega = 2\pi f$ the angular frequency. For a given angular frequency, only modes such that κ_z^2 is positive can propagate. The cutoff frequency of each mode is calculated as

$$f_j = \frac{\sqrt{\omega^2 - c^2 \kappa_z^2}}{2\pi}$$

Results and Discussion

The model finds five propagating modes, whose characteristics are summed up in the table here below.

Cutoff frequency (Hz)	Characteristics	
0	Plane wave	
635	Antisymmetric with respect to x, symmetric with respect to y	
1210	Symmetric with respect to \mathbf{x} , antisymmetric with respect to \mathbf{y}	
1240	Symmetric with respect to x and y	
1467	Antisymmetric with respect to x and y	

For a muffler with a centered tube leading into the chamber, the first mode that is symmetric with respect to both the *x*-axis and the *y*-axis is propagating when the frequency is higher than 1240 Hz. Figure 2 shows this mode, which for an infinitely long chamber occurs at 1240 Hz.



Figure 2: The chamber's first fully symmetric propagation mode. The plot shows the real part of the acoustic pressure.

Application Library path: Acoustics Module/Automotive/

eigenmodes_in_muffler

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click 🚳 Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **9** 2D.
- 2 In the Select Physics tree, select Acoustics>Pressure Acoustics>Pressure Acoustics, Frequency Domain (acpr).

- 3 Click Add.
- 4 Click 🔿 Study.
- 5 In the Select Study tree, select Preset Studies for Selected Physics Interfaces> Mode Analysis.
- 6 Click **M** Done.

GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.

Square 1 (sq1)

- I In the **Geometry** toolbar, click **Square**.
- 2 In the Settings window for Square, locate the Size section.
- **3** In the **Side length** text field, type 150.
- 4 Locate the **Position** section. From the **Base** list, choose **Center**.

Circle I (c1)

- I In the **Geometry** toolbar, click \bigcirc **Circle**.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type 75.
- 4 Locate the **Position** section. In the **x** text field, type -75.

Circle 2 (c2)

- I In the **Geometry** toolbar, click \bigcirc **Circle**.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- **3** In the **Radius** text field, type **75**.
- 4 Locate the **Position** section. In the **x** text field, type **75**.

Union I (unil)

- I In the Geometry toolbar, click i Booleans and Partitions and choose Union.
- 2 Click in the Graphics window and then press Ctrl+A to select all objects.
- 3 In the Settings window for Union, locate the Union section.
- 4 Clear the Keep interior boundaries check box.
- 5 Click 📑 Build All Objects.
- 6 Click the \leftrightarrow Zoom Extents button in the Graphics toolbar.

ADD MATERIAL

- I In the Home toolbar, click 🙀 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

By default, the boundaries of the geometry will be considered to be sound hard walls. No other physics settings are needed.

MESH I

In this model, the mesh is set up manually. Add a free triangular mesh and then proceed to the solver settings.

Free Triangular 1 In the Mesh toolbar, click K Free Triangular.

STUDY I

Step 1: Mode Analysis

- I In the Model Builder window, under Study I click Step I: Mode Analysis.
- 2 In the Settings window for Mode Analysis, locate the Study Settings section.
- 3 Select the Desired number of modes check box. In the associated text field, type 8.
- 4 Select the Search for modes around check box. In the associated text field, type 20.

The free-space propagation mode has an out-of-plane wave number equal to omega/ c = 27.5 rad/m. With these settings, the solver returns the 8 modes with propagation constants closest to 20 rad/m first in the list.

5 In the Mode analysis frequency text field, type 1500[Hz].

This setting makes the software look for propagating modes with cutoff frequencies up to 1500 Hz.

6 In the Home toolbar, click **=** Compute.

RESULTS

Acoustic Pressure (acpr)

The solver has found the free-space mode and all other propagating modes. There is a total of 5 different propagating modes. Because the waves can propagate both into and out of

the modeling plane, each mode gets reported twice, with positive and negative out-ofplane wave numbers.

For the positive out-of-plane wave numbers, it holds that the higher the mode, the lower the wave number. However, the solver does not stop at zero. Because you asked for more than the 5 existing propagating modes, you get additional modes with imaginary out-of-plane wave numbers. This indicates that they are evanescent. The default plot shows the acoustic pressure distribution for a mode with a wave number of -17.95i rad/m.

- I In the Settings window for 2D Plot Group, locate the Data section.
- 2 From the Out-of-plane wave number (rad/m) list, choose 15.461.
- 3 In the Acoustic Pressure (acpr) toolbar, click 💿 Plot.
- **4** Click the **Com Extents** button in the **Graphics** toolbar.

This is the lowest fully symmetric mode, which is shown in Figure 2. You can compute the cutoff frequency of this mode using the expression in the model introduction. In order to refer to the speed of sound in air, use an arbitrary point in the geometry for this evaluation.

Point Evaluation 1

- I In the Results toolbar, click $\frac{8.85}{e-12}$ Point Evaluation.
- 2 In the Settings window for Point Evaluation, locate the Data section.
- 3 From the Out-of-plane wave number selection list, choose From list.
- 4 In the Out-of-plane wave number (rad/m) list, select 15.461.
- **5** Select Point 3 only.
- 6 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
<pre>sqrt(acpr.omega^2-acpr.kz^2*acpr.c^2)/(2*pi)</pre>	rad/s	

7 Click **= Evaluate**.

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