

Acoustics of a Particulate-Filter-Like System

Introduction

This is a model of the acoustics in a particulate-filter-like system. Real systems, like diesel particulate filters (DPFs), are designed to remove/filter soot (diesel particles) from the exhaust of diesel engine vehicles. The porous medium in such systems are typically structured with long air-filled ducts. To simplify this model, the filter geometry is assumed to be axisymmetric and the ducts are represented by long cylindrical groves inside a porous material plug. Although the main function of a particulate filter is filtering of the exhaust flow, the filter also has acoustic damping properties that relate to the muffler system.

The model analyzes the acoustic properties of the simplified 2D axisymmetric particulatefilter like geometry using the Acoustic-Poroelastic Waves Interaction multiphysics interface. This interface describes the small-deformation elastic waves propagating in a porous material coupled to waves in a fluid by an Acoustic-Porous Boundary multiphysics coupling. The model accounts for the coupled displacement and is thus a fluid-structure interaction problem.

Model Definition

Three aligned cylinders make up the particulate filter system under study: an inlet, an outlet, and a main filter cylinder. The particulate filter is located inside the filter cylinder. Figure 1 shows a sketch of a cross section in the *rz*-plane of the 2D axisymmetric geometry. The filter in the central region is of length $L_{\text{filter}} = 200 \text{ mm}$ with a filter radius of $R_{\text{filter}} = 150 \text{ mm}$. The inlet and outlet pipe radii are $R_{\text{tube}} = 50 \text{ mm}$. The filter consists of a structured air-filled porous material (the brown region), which could be a silicon carbide matrix. The air-filled groves (light blue) have a width of $d_{\text{h}} = 5 \text{ mm}$ and the porous walls are of thickness $h_{\text{t}} = 3.2 \text{ mm}$. At the end of each grove there is an impermeable steel plug (black). The rest of the system is filled with air.

The wide groves are used to simplify the model. In real DPF systems the groves are replaced by long slender ducts with a typical width of 1-2 mm and the porous walls have a typical width of 0.3-0.5 mm.



Figure 1: Geometry of the particulate filter with dimensions indicated.

The porous material is assumed to be isotropic with the material parameters as listed in Table 1.

PARAMETER	VALUE	DESCRIPTION
Ε	20 GPa	Young's modulus
ν	0.4	Poisson's ratio
ρ _d	1000 kg/m ³	Drained matrix density
$\alpha_{\rm B}$	0.3	Biot-Willis coefficient
ε _p	0.3	Porosity
κ _p	10 ⁻¹¹ m ²	Permeability of porous matrix
τ	I	Tortuosity factor

TABLE I: MATERIAL PARAMETERS OF THE POROUS MATRIX.

Note that the Biot-Willis coefficient is equal to the porosity for rigid porous materials and is equal to 1 for a soft porous material (or a suspension of solid in liquid). The fluid parameters are those of air including the compressibility, χ , which for an ideal gas is equal to $(p_0)^{-1}$, where p_0 is the absolute pressure (here 1 atm).

The filter is characterized acoustically by the transmission loss, TL (given in dB), as a function of the frequency, f. It is defined as

$$TL(f) = 20\log\left(\frac{p_{\text{incident}}}{p_{\text{out}}}\right)$$

where p_{incident} is the incident inlet pressure and p_{out} is the outlet pressure. You solve the model for the frequency interval 20 Hz–2000 Hz.

When setting up the porous material model, you also need to specify whether to use the low-frequency (default) or high-frequency range approximation for the fluid viscosity. The transition between the two ranges is defined by the reference frequency f_c given by the expression

$$f_c = \frac{\varepsilon_p \mu}{2\pi\kappa\rho_f}$$

where ρ_f is the fluid density (for air 1.2 kg/m³) and μ is the dynamic viscosity of the fluid (for air 1.8·10⁻⁵ Pa·s). Using the above material parameters gives a reference frequency of the order 100 kHz. Thus, the low-frequency range applies to the current problem.

Results and Discussion

The acoustic transmission loss TL through the axisymmetric simplified particulate filter is determined for the frequency range 20 Hz to 2000 Hz and depicted in Figure 2.



Figure 2: Transmission through the simple particulate filter as a function of frequency.

The loss is seen to be of the same order of magnitude as in real particulate filters (like diesel particulate filters, DPFs) the porous medium is often, as mentioned, structured with long ducts that decrease the acoustic damping while retaining good filtering properties. In this axisymmetric model the ducts take the form of cylindrical slits in 3D, which may introduce some nonstandard resonances in the filter. Moreover, in a real exhaust system there is an interaction between the exhaust flow and the acoustics (here the no-flow situation is studied), and the temperature is higher than 20 °C (as used here). Other physical effects include acoustic-structure and poroelastic-structure interactions with the exhaust pipe system. The present simplified model enables isolating the acoustics problem from other physical phenomena.

Figure 3 depicts the pressure distribution inside the particulate filter model for 20 Hz and for 2 kHz.



Figure 3: Pressure distribution inside the particulate filter for f = 20 Hz (top) and f = 2 kHz (bottom).

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Application Library path: Acoustics_Module/Automotive/ acoustics_particulate_filter

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click 🔗 Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click 🚈 2D Axisymmetric.
- 2 In the Select Physics tree, select Acoustics>Acoustic-Structure Interaction>Acoustic-Solid-Poroelastic Waves Interaction.
- 3 Click Add.
- 4 Click 🔿 Study.
- 5 In the Select Study tree, select General Studies>Frequency Domain.
- 6 Click 🗹 Done.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click **b** Load from File.
- 4 Browse to the model's Application Libraries folder and double-click the file acoustics_particulate_filter_parameters.txt.

The parameters loaded here define the geometric dimensions and the tortuosity parameter used in the poroelastic model. Because the geometry is now parameterized, changing the dimensions in the parameters list will update the geometry automatically.

GEOMETRY I

Rectangle 1 (r1)

I In the Geometry toolbar, click 📃 Rectangle.

- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type Rtube.
- 4 In the **Height** text field, type Ltube.

Rectangle 2 (r2)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type Rfilter.
- 4 In the **Height** text field, type Lair.
- **5** Locate the **Position** section. In the **z** text field, type Ltube.

Rectangle 3 (r3)

- I In the **Geometry** toolbar, click **Rectangle**.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type Rfilter.
- 4 In the Height text field, type Lfilter.
- 5 Locate the **Position** section. In the z text field, type Ltube+Lair.
- 6 Click to expand the Layers section. In the table, enter the following settings:

Layer name	Thickness (m)		
Layer 1	ht		

7 Select the Layers on top check box.

Rectangle 4 (r4)

- I In the **Geometry** toolbar, click **Rectangle**.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type Rfilter.
- 4 In the **Height** text field, type Lair.
- 5 Locate the **Position** section. In the z text field, type Ltube+Lair+Lfilter.

Rectangle 5 (r5)

- I In the Geometry toolbar, click 📃 Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type Rtube.
- 4 In the **Height** text field, type Ltube.

5 Locate the **Position** section. In the z text field, type Ltube+2*Lair+Lfilter.

Rectangle 6 (r6)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type dh.
- 4 In the **Height** text field, type Lfilter.
- **5** Locate the **Position** section. In the **r** text field, type ht.
- 6 In the z text field, type Ltube+Lair.

Array I (arr I)

- I In the Geometry toolbar, click 💭 Transforms and choose Array.
- 2 Select the object r6 only.
- 3 In the Settings window for Array, locate the Size section.
- 4 In the **r size** text field, type 18.
- **5** Locate the **Displacement** section. In the **r** text field, type (dh+ht).

Form Union (fin)

- I In the Model Builder window, click Form Union (fin).
- 2 In the Settings window for Form Union/Assembly, click 📳 Build Selected.

3 Click the \longleftrightarrow **Zoom Extents** button in the **Graphics** toolbar.

The geometry of the diesel particulate filter should look like the figure below.



DEFINITIONS

All domains

- I In the **Definitions** toolbar, click **The Box**.
- 2 In the Settings window for Box, type All domains in the Label text field.

Poroelastic Waves Domains

- I In the **Definitions** toolbar, click http://www.explicit.
- 2 In the Settings window for Explicit, type Poroelastic Waves Domains in the Label text field.
- **3** Select Domains 3–5, 11–13, 17–19, 23–25, 29–31, 35–37, 41–43, 47–49, 53–55, 59–61, 65–67, 71–73, 77–79, 83–85, 89–91, 95–97, 101–103, 107–109, and 113–115 only.

Steel Domains

- I In the **Definitions** toolbar, click http://www.click
- 2 In the Settings window for Explicit, type Steel Domains in the Label text field.

3 Select Domains 10, 14, 22, 26, 34, 38, 46, 50, 58, 62, 70, 74, 82, 86, 94, 98, 106, and 110 only.

Air Domains

- I In the **Definitions** toolbar, click 🛅 **Difference**.
- 2 In the Settings window for Difference, type Air Domains in the Label text field.
- 3 Locate the Input Entities section. Under Selections to add, click + Add.
- 4 In the Add dialog box, select All domains in the Selections to add list.
- 5 Click OK.
- 6 In the Settings window for Difference, locate the Input Entities section.
- 7 Under Selections to subtract, click + Add.
- 8 In the Add dialog box, in the Selections to subtract list, choose Poroelastic Waves Domains and Steel Domains.
- 9 Click OK.

ADD MATERIAL

- I In the Home toolbar, click 🙀 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component in the window toolbar.

By default, the first material node applies to all domains except where overridden by subsequent nodes in the Materials branch.

Next, create a poroelastic material with user-defined material parameters for the solid matrix.

MATERIALS

SiC matrix

- I In the Model Builder window, under Component I (comp1) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type SiC matrix in the Label text field.

The material properties for the SiC matrix will be defined after the physics interface settings.

ADD MATERIAL

I Go to the Add Material window.

- 2 In the tree, select Built-in>Steel AISI 4340.
- 3 Click Add to Component in the window toolbar.
- 4 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

MATERIALS

Steel AISI 4340 (mat3)

- I In the Settings window for Material, locate the Geometric Entity Selection section.
- 2 From the Selection list, choose Steel Domains.

The steel plugs are the domains selected in the figure below.



POROELASTIC WAVES (PELW)

- I In the Model Builder window, under Component I (compl) click Poroelastic Waves (pelw).
- 2 In the Settings window for Poroelastic Waves, locate the Domain Selection section.
- 3 From the Selection list, choose Poroelastic Waves Domains.

Having defined the materials, you can specify the domain settings.

Poroelastic Material I

I In the Model Builder window, under Component I (compl)>Poroelastic Waves (pelw) click Poroelastic Material I.

- **2** In the Settings window for Poroelastic Material, locate the Porous Matrix Properties section.
- 3 From the Porous elastic material list, choose SiC matrix (mat2).
- 4 From the Specify list, choose Young's modulus and Poisson's ratio.

Use a linear elastic material model for the steel-plug domains.

SOLID MECHANICS (SOLID)

- I In the Model Builder window, under Component I (compl) click Solid Mechanics (solid).
- 2 In the Settings window for Solid Mechanics, locate the Domain Selection section.
- **3** From the **Selection** list, choose **Steel Domains**.

POROELASTIC WAVES (PELW)

In the Model Builder window, under Component I (compl) click Poroelastic Waves (pelw).

Fixed Constraint I

I In the Physics toolbar, click — Boundaries and choose Fixed Constraint.

2 Select Boundaries 273–275 only.

The porous matrix is assumed to be glued to an outer casing at the boundary highlighted in the figure below; hence the fixed constraint boundary condition. This boundary is also sound hard.



PRESSURE ACOUSTICS, FREQUENCY DOMAIN (ACPR)

- In the Model Builder window, under Component I (compl) click Pressure Acoustics, Frequency Domain (acpr).
- **2** In the Settings window for Pressure Acoustics, Frequency Domain, locate the Domain Selection section.

3 From the Selection list, choose Air Domains.

The selected air domain where Pressure Acoustics apply is depicted in the figure below. It consists of the inlet and outlet as well as the thin air groves inside the particulate filter.



Port I



- **2** Select Boundary 2 only.
- 3 In the Settings window for Port, locate the Port Properties section.
- **4** From the **Type of port** list, choose **Circular**.
- 5 Locate the **Incident Mode Settings** section. In the A^{in} text field, type p0.

Port 2

- I In the Physics toolbar, click Boundaries and choose Port.
- 2 Select Boundary 15 only.
- 3 In the Settings window for Port, locate the Port Properties section.
- **4** From the **Type of port** list, choose **Circular**.

You have now specified the domain settings. The red cross decoration for the Air and the SiC matrix nodes under Materials indicates that there are still undefined material

parameters. To discover what is missing, return to the Materials branch before proceeding with the boundary conditions.

MATERIALS

Air (mat1)

- I In the Model Builder window, under Component I (compl)>Materials click Air (matl).
- 2 In the Settings window for Material, locate the Material Contents section.
- **3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Compressibility of fluid	chif	1/1[atm]	I/Pa	Basic

Recall that the compressibility of an ideal gas at the pressure P0 equals 1/P0.

SiC matrix (mat2)

- I In the Model Builder window, click SiC matrix (mat2).
- 2 In the Settings window for Material, locate the Material Contents section.
- **3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Young's modulus	E	20[GPa]	Pa	Young's modulus and Poisson's ratio
Poisson's ratio	nu	0.4	1	Young's modulus and Poisson's ratio
Density	rho	1000	kg/m³	Basic
Porosity	epsilon	0.3	I	Basic
Permeability	kappa_iso ; kappaii = kappa_iso, kappaij = 0	1e-11	m²	Basic
Biot-Willis coefficient	alphaB	0.3	I	Poroelastic material
Tortuosity factor	tau	tauP	I	Poroacoustics model

MULTIPHYSICS

The coupling between the acoustic and the porous domain, and between the acoustic and the elastic domain is automatically set up under the **Multiphysics** node. Click on the node and then inspect the **Acoustic-Porous Boundary I** node and the **Acoustic-Structure Boundary**

I node. Both nodes have the selection automatically set to **All boundaries** - this default setting automatically couples all relevant boundaries.

MESH I

In this model, the mesh is set up manually. Proceed by directly adding the desired mesh component.

Use a mapped mesh with boundary layers added to resolve the continuity condition between the poroelastic and pressure acoustic domains.

Mapped I

In the Mesh toolbar, click Mapped.

Size

- I In the Model Builder window, click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Predefined list, choose Extra fine.

Mapped I

- I In the Model Builder window, click Mapped I.
- 2 In the Settings window for Mapped, click to expand the Reduce Element Skewness section.
- 3 Select the Adjust edge mesh check box.

Size 1

- I Right-click Mapped I and choose Size.
- 2 In the Settings window for Size, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Domain.

Select all the domains that make up the particulate filter, for easy selection use the **Select Box** utility.



4 From the Selection list, choose Poroelastic Waves Domains.

- 5 Locate the Element Size section. Click the Custom button.
- 6 Locate the Element Size Parameters section.
- 7 Select the Maximum element size check box. In the associated text field, type ht.
- 8 Click 📗 Build All.

Boundary Layers 1

- I In the Mesh toolbar, click Moundary Layers.
- 2 In the Settings window for Boundary Layers, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domains 2 and 6 only.
- **5** Click to expand the **Transition** section. Clear the **Smooth transition to interior mesh** check box.

Boundary Layer Properties

Now, select all the boundaries between the particulate filter and the air, for easy selection use the **Select Box** utility. You can also paste in the numbers given below.

I In the Model Builder window, click Boundary Layer Properties.

2 Select Boundaries 6, 12, 17, 22, 24, 29, 31, 36, 38, 43, 45, 50, 52, 57, 59, 64, 66, 71, 73, 78, 80, 85, 87, 92, 94, 99, 105, 110, 112, 117, 119, 124, 126, 131, 133, 138, 140, 145, 147, 152, 154, 159, 161, 166, 168, 173, 175, 180, 182, 187, 189, 194, 196, 201, 203, 208, 210, 215, 217, 222, 224, 229, 231, 236, 238, 243, 245, 250, 252, 257, 259, 264, 266, and 271 only.



3 In the Settings window for Boundary Layer Properties, locate the Layers section.

- 4 In the Number of layers text field, type 4.
- 5 From the Thickness specification list, choose First layer.
- 6 In the Thickness text field, type ht/2.

7 Click 📗 Build All.

When modeling poroelastic waves coupled to pressure acoustics, it is good practice to include a boundary layer mesh on the interface between the two physics. This is due to the fact that large gradients in the dependent variables appear here.



The mesh should look like the figure below.

STUDY I

Step 1: Frequency Domain

I In the Model Builder window, under Study I click Step I: Frequency Domain.

- 2 In the Settings window for Frequency Domain, locate the Study Settings section.
- 3 In the Frequencies text field, type range(20,20,2000).
- **4** In the **Home** toolbar, click **= Compute**.

RESULTS

Surface 2

I In the Model Builder window, right-click Displacement (pelw) and choose Surface.

- 2 In the Settings window for Surface, locate the Expression section.
- **3** In the **Expression** text field, type solid.disp.

- 4 Click to expand the Inherit Style section. From the Plot list, choose Surface.
- 5 Click to expand the Title section. From the Title type list, choose None.

Deformation I

I Right-click Surface 2 and choose Deformation.

The plot should now look like the figure below. It depicts the displacement magnitude.



Surface 2

- I In the Model Builder window, right-click Displacement, 3D (pelw) and choose Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type solid.disp.
- 4 Click to expand the Inherit Style section. From the Plot list, choose Surface.
- 5 Click to expand the Title section. From the Title type list, choose None.

Deformation I

- I Right-click Surface 2 and choose Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- 3 In the **R-component** text field, type cos(rev1phi)*u2.
- 4 In the **PHI-component** text field, type sin(rev1phi)*v2.

Displacement, 3D (pelw)

I Click the \longleftrightarrow Zoom Extents button in the Graphics toolbar.

Displacement in the porous matrix at 2 kHz, is depicted in the figure below.



- 2 In the Model Builder window, under Results click Displacement, 3D (pelw).
- 3 In the Settings window for 3D Plot Group, locate the Data section.
- 4 From the Parameter value (freq (Hz)) list, choose 20.

5 In the Displacement, 3D (pelw) toolbar, click 💽 Plot.

Depicted below, the displacement magnitude in the porous matrix after changing the evaluation frequency to 20 Hz.



Sound Pressure Level (acpr)

I In the Model Builder window, click Sound Pressure Level (acpr).

2 In the Sound Pressure Level (acpr) toolbar, click **O** Plot.

The sound pressure level in dB (relative to 20 μ Pa) at 2 kHz, is depicted below.



Acoustic Pressure, 3D (acpr)

- I In the Model Builder window, click Acoustic Pressure, 3D (acpr).
- 2 In the Acoustic Pressure, 3D (acpr) toolbar, click 💿 Plot.

The plot should look like the one depicted in Figure 3 (bottom).

- 3 In the Settings window for 3D Plot Group, locate the Data section.
- 4 From the Parameter value (freq (Hz)) list, choose 20.
- 5 In the Acoustic Pressure, 3D (acpr) toolbar, click 🗿 Plot.

The plot, now evaluated at 20 Hz, should look like the one depicted in Figure 3 (top).

SPL at inlet and outlet

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type SPL at inlet and outlet in the Label text field.

Octave Band I

- I In the SPL at inlet and outlet toolbar, click \sim More Plots and choose Octave Band. Select boundary as the geometric entity and the pressure is automatically averaged on that boundary.
- 2 In the Settings window for Octave Band, locate the Selection section.
- **3** From the Geometric entity level list, choose Boundary.
- **4** Select Boundary 15 only.
- 5 Locate the Plot section. From the Quantity list, choose Continuous power spectral density.Change the style to octave or 1/3 octave if you like the SPL given in bands.
- 6 Click to expand the Legends section. Select the Show legends check box.
- 7 From the Legends list, choose Manual.
- 8 In the table, enter the following settings:

Legends

SPL at outlet

Octave Band 2

I Right-click Octave Band I and choose Duplicate.

Change the selection for the boundary.

- 2 In the Settings window for Octave Band, locate the Selection section.
- **3** Click to select the **Delta Activate Selection** toggle button.
- 4 Click K Clear Selection.
- **5** Select Boundary 2 only.
- 6 Locate the Legends section. In the table, enter the following settings:

Legends

SPL at inlet

SPL at inlet and outlet

- I In the Model Builder window, click SPL at inlet and outlet.
- 2 In the Settings window for ID Plot Group, locate the Legend section.
- 3 From the **Position** list, choose **Lower right**.
- 4 Locate the Axis section. Clear the x-axis log scale check box.

5 In the SPL at inlet and outlet toolbar, click **I** Plot.

The plot should look like the figure below. It depicts the sound pressure level in dB (relative to μ Pa) at the inlet (blue line) and the outlet (green line).



Transmission Loss

- I In the Home toolbar, click 📠 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Transmission Loss in the Label text field.

Octave Band I

- I In the Transmission Loss toolbar, click \sim More Plots and choose Octave Band.
- 2 In the Settings window for Octave Band, locate the Selection section.
- 3 From the Geometric entity level list, choose Global.
- 4 Locate the y-Axis Data section. In the Expression text field, type acpr.port1.P_in.
- 5 From the **Expression type** list, choose **Power**.
- **6** In the **Power reference** text field, type acpr.port2.P_out.
- 7 Locate the Plot section. From the Quantity list, choose Continuous power spectral density.

Transmission Loss

I In the Model Builder window, click Transmission Loss.

- 2 In the Settings window for ID Plot Group, click to expand the Title section.
- **3** From the **Title type** list, choose **None**.
- 4 Locate the **Plot Settings** section.
- 5 Select the **y-axis label** check box. In the associated text field, type Transmission loss (dB).
- 6 Locate the Axis section. Clear the x-axis log scale check box.
- 7 In the Transmission Loss toolbar, click **O** Plot.

The plot should look like the one depicted in Figure 2.