

Acoustic Cloaking

Recent studies (see Ref. 1) have shown the feasibility of manufacturing "invisibility cloaks" using layered metamaterials. Draping an object in a cloak makes it transparent, or nearly transparent, to electromagnetic waves.

The same principle can be used in acoustics to hide an object from acoustic radiation. This example looks at sound scattering from a plane wave incident on a hard-walled cylinder. Results with the cylinder clad in a homogenized cloak and a layered cloak (described in Ref. 2) are compared to a solution without the cloak.

Model Definition

A cylinder with a 1 m radius is surrounded by a metamaterial cloak consisting of a fluid with anisotropic density and scalar bulk modulus. Such a fluid can be modeled with the built-in **Anisotropic Acoustics** material model.

A way to set up an approximation to such a model is to use layers of two alternating fluids. In this tutorial, we use a 50 layer model with a layer thickness of 2 cm and a 20 layer model with a layer thickness of 5 cm. The material properties are similar to those in Ref. 2 and defined as follows:

$$\begin{split} \rho_1 &= \frac{r + \sqrt{2rR_1 - R_1^2}}{r - R_1} \rho_b \\ c_1 &= \frac{R_2 - R_1}{R_2} \frac{r}{r - R_1} c_b \\ \rho_2 &= \rho_b^2 / \rho_1 \\ c_2 &= c_1 \end{split}$$

In this formula:

- ρ_1 , ρ_2 , c_1 , and c_2 are the density and speed of sound of materials 1 and 2;
- $\rho_b = 1.25 \text{ kg/m}^3$ and $c_b = 343 \text{ m/s}$, are the density and speed of sound in the outside medium, which is air;
- R_1 and R_2 are the inner and outer radius of the cloak; and
- r is the distance to the cylinder axis.

The material data for the corresponding anisotropic material can be calculated in the homogenization limit: for the effective bulk modulus and the effective density in the tangential direction (along the layers) as the volume average of their reciprocals; and for the effective density in the normal direction (perpendicular to the layers) as the volume average.

$$K = \frac{2K_1K_2}{K_1 + K_2}$$

$$\rho_t = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$$

$$\rho_n = \frac{\rho_1 + \rho_2}{2}$$

The model considers a frequency of f = 300 Hz and solves the Helmholtz equation for the total acoustic pressure:

$$\nabla \cdot (-\rho^{-1} \nabla p_{t}) - \frac{\omega^{2} p_{t}}{K} = 0$$

Here, ρ^{-1} is a tensor for the anisotropic material, the bulk modulus $K = \rho c^2$ for the isotropic material, and p_t is the total acoustic pressure. To describe an incident plane wave traveling in the x direction, a background field p_b is defined as e^{-ik_bx} , where $k_b = 2\pi f/c_b$ is the propagation constant in the background medium. The equation is solved for the scattered field p_s , using the definition

$$p_t \equiv p_b + p_s$$

The geometric mirror symmetry of the problem is used to reduce the modeling domain to half of the full geometry (see Figure 1). The background medium is truncated with a cylindrical radiation condition (see the Acoustics Module User's Guide for details about the theory) on its outer boundaries.

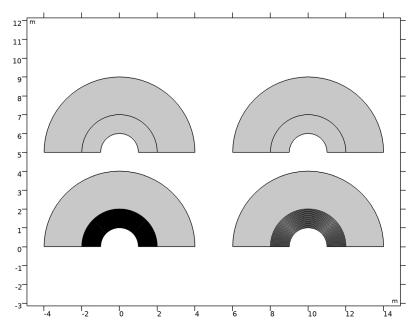


Figure 1: The geometries used in the tutorial: homogenized cloak (upper left), no cloak (upper right), 50 layer cloak (lower left), and 20 layer cloak (lower right). The center cylinder cut out from the geometry is covered by the cloak and outside the cloak the background material is truncated at a 4 m distance from the center. The layers in the 50 layer cloak geometry are too thin to see clearly in this image.

Results and Discussion

The total acoustic pressure for the four different cases can be seen in Figure 2. The top right figure shows the pressure field without the cloak, when the cylinder is surrounded only by air. The incident pressure wave is scattered in all directions and is significantly influenced by the cylinder. In the upper-left figure, we see the homogenized cloak in use. The incident wave is undisturbed outside the cloak and it is not possible to determine that there is a cylinder present at all. The two bottom figures show how the cloak gets better when the number of layers is increased and the model is more similar to the homogenized cloak.

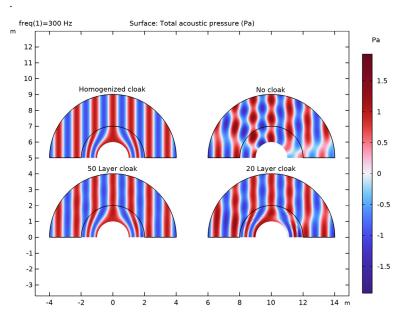


Figure 2: Total acoustic pressure for the four studied cases: homogenized cloak (upper left), no cloak (upper right), 50 layer cloak (lower left), and 20 layer cloak (lower right).

Figure 3 shows the total sound pressure level for the same cases. When no cloak is used, the shadow zone behind the cylinder is easily visible as well as the pressure peaks on the side where the wave is incident. With the homogenized cloak, the pressure variations are not visible.

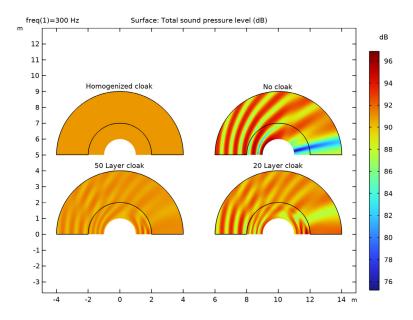


Figure 3: Total sound pressure level for the four studied cases: homogenized cloak (upper left), no cloak (upper right), 50 layer cloak (lower left), and 20 layer cloak (lower right).

Looking only at the scattered sound pressure level (Figure 4) we can see a significant difference in the scattered field for the four cases. The scattered sound pressure level decreases as the cloak tends toward the homogenized cloak.

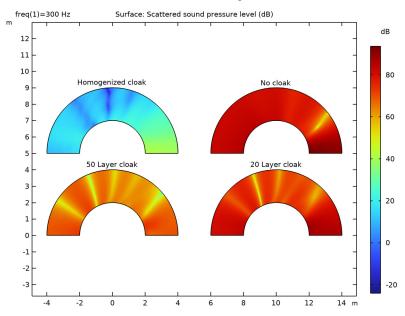


Figure 4: Scattered sound pressure level for the four studied cases: homogenized cloak (upper left), no cloak (upper right), 50 layer cloak (lower left), and 20 layer cloak (lower right).

Another way to illustrate the effect of the cloak is to look at the total acoustic pressure along the cloak boundary. This is shown in Figure 5, where we can see that the background pressure field curve coincides with the curve for the homogenized cloak as expected for an effective cloak.

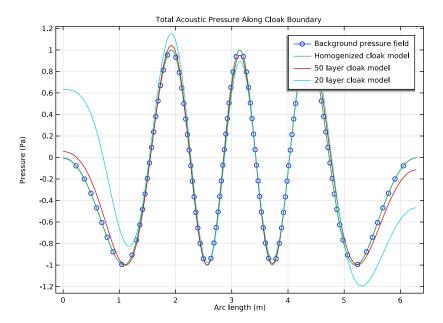


Figure 5: Pressure at the acoustic cloak boundary.

Finally, in Figure 6, the speed of sound in the principal directions of the homogenized anisotropic material is shown.

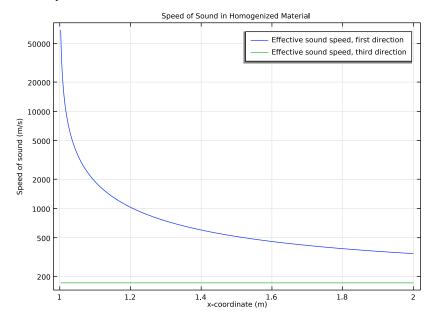


Figure 6: Speed of sound across the acoustic cloak.

The instructions and results that follow, deal only with a frequency of 300 Hz, but you can modify the model and use a different frequency, or sweep over a range of frequencies, to see how this affects the cloaking.

References

1. Several COMSOL Conference papers:

https://www.comsol.com/paper/5478/ https://www.comsol.com/paper/9831/

https://www.comsol.com/paper/5463/

2. D. Torrent and J. Sánchez-Dehesa, "Acoustic Cloaking in Two Dimensions: a Feasible Approach," New Journal of Physics, vol. 10, 063015, 2008.

Application Library path: Acoustics_Module/Tutorials,_Pressure_Acoustics/ acoustic_cloaking

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **2** 2D.
- 2 In the Select Physics tree, select Acoustics>Pressure Acoustics, Frequency Domain (acpr).
- 3 Click Add.
- 4 Click Study.
- 5 In the Select Study tree, select General Studies>Frequency Domain.
- 6 Click M Done.

GEOMETRY I

The geometry of this model is simple but a bit repetitive to create, due to the large number of circles involved. You will therefore import a geometry sequence from a file. The instructions for building the geometry can be found in the appendix at the end of this document.

- I In the Geometry toolbar, click Insert Sequence and choose Insert Sequence.
- **2** Browse to the model's Application Libraries folder and double-click the file acoustic_cloaking_geom_sequence.mph.
- 3 In the Geometry toolbar, click **Build All**.
- 4 Click the Zoom Extents button in the Graphics toolbar.

The geometry should look like that in Figure 1.

The file you have just imported contains a few parameters that are used to generate the geometry. Now proceed to add some additional parameters that will be used in the analysis.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
rhob	1.25[kg/m^3]	1.25 kg/m³	Density, background material
cb	343[m/s]	343 m/s	Speed of sound, background material
f0	300[Hz]	300 Hz	Frequency of the analysis
lam0	cb/f0	1.1433 m	Wavelength

The material properties in the cloaking involve dependencies on the inner and outer radius, the local distance from the center, and the properties of the surrounding background fluid. It is convenient to define these expressions as variables.

DEFINITIONS

Cylindrical System 2 (sys2)

- I In the Definitions toolbar, click \bigvee_{x}^{z} Coordinate Systems and choose Cylindrical System.
- 2 In the Settings window for Cylindrical System, locate the Settings section.
- **3** Find the **Origin** subsection. In the table, enter the following settings:

x (m)	y (m)
x1	y1

Radial Coordinate: Homogenized Cloak

- I In the Model Builder window, right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, type Radial Coordinate: Homogenized Cloak in the Label text field.
- 3 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- 4 From the Selection list, choose Selection: Homogenized Cloak.

5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
r	sys2.r	m	Radial coordinate

Radial Coordinate: 50 Layer Cloak

- I Right-click **Definitions** and choose **Variables**.
- 2 In the Settings window for Variables, type Radial Coordinate: 50 Layer Cloak in the Label text field.
- 3 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- 4 From the Selection list, choose Selection: 50 Layer Cloak.
- **5** Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
r	sqrt((x-x3)^2+(y-y3)^2)	m	Radial coordinate

Radial Coordinate: 20 Layer Cloak

- I Right-click **Definitions** and choose **Variables**.
- 2 In the Settings window for Variables, type Radial Coordinate: 20 Layer Cloak in the Label text field.
- 3 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- 4 From the Selection list, choose Selection: 20 Layer Cloak.
- **5** Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
r	sqrt((x-x4)^2+(y-y4)^2)	m	Radial coordinate

Variables: Acoustic Cloak Data

- I In the **Definitions** toolbar, click a= **Local Variables**.
- 2 In the Settings window for Variables, type Variables: Acoustic Cloak Data in the Label text field.
- 3 Locate the Geometric Entity Selection section. From the Geometric entity level list, choose Domain.
- 4 From the Selection list, choose Selection: Acoustic Cloak.

5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
rho1	rhob*(r+sqrt(2*r* R1-R1^2))/(r-R1)	kg/m³	Density, Material 1
c1	cb*(R2-R1)/R2*r/ (r-R1)	m/s	Speed of sound, Material 1
rho2	rhob^2/rho1	kg/m³	Density, Material 2
c2	c1	m/s	Speed of sound, Material 2
K1	rho1*c1^2	Pa	Bulk modulus, Material 1
K2	rho2*c2^2	Pa	Bulk modulus, Material 2
К	2*K1*K2/(K1+K2)	Pa	Effective bulk modulus
rho_tangential	2*rho1*rho2/(rho1+ rho2)	kg/m³	Density along layers
rho_normal	(rho1+rho2)/2	kg/m³	Density perpendicular to the layers

Now, proceed to add the materials. Material 1 and Material 2 are the two materials with varying properties that will be used in the layers to create an acoustic cloak. The homogenized material is an anisotropic material which properties also vary with thickness and are equivalent to a model with a large number of layers.

MATERIALS

Air

- I In the Model Builder window, under Component I (comp I) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Air in the Label text field.
- **3** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rhob	kg/m³	Basic
Speed of sound	С	cb	m/s	Basic

Material I

I Right-click Materials and choose Blank Material.

- 2 In the Settings window for Material, type Material 1 in the Label text field.
- 3 Locate the Geometric Entity Selection section. From the Selection list, choose Selection: Material I.
- **4** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho1	kg/m³	Basic
Speed of sound	С	c1	m/s	Basic

Material 2

- I Right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Material 2 in the Label text field.
- 3 Locate the Geometric Entity Selection section. From the Selection list, choose Selection: Material 2.
- **4** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho2	kg/m³	Basic
Speed of sound	с	c2	m/s	Basic

Homogenized Material

- I Right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Homogenized Material in the Label text field.
- 3 Locate the Geometric Entity Selection section. From the Selection list, choose Selection: Homogenized Cloak.
- 4 Click to expand the Material Properties section. In the Material properties tree, select Acoustics>Anisotropic Acoustics Model>Effective bulk modulus (K_eff).
- 5 Click + Add to Material.
- 6 In the Material properties tree, select Acoustics>Anisotropic Acoustics Model> Effective density (rho_eff).

7 Locate the Material Contents section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Effective density	{rho_eff11, rho_eff22, rho_eff33}; rho_effij = 0	{rho_normal, rho_tangential, rho_tangential}	kg/m³	Anisotropic acoustics model
Effective bulk modulus	K_eff	K	Pa	Anisotropic acoustics model
Density	rho		kg/m³	Basic
Speed of sound	С		m/s	Basic

The homogenized material has a diagonal density matrix with terms that change value with the radius. The bulk modulus also varies with the radius.

PRESSURE ACOUSTICS, FREQUENCY DOMAIN (ACPR)

Homogenized Model

- I In the Model Builder window, under Component I (compl) right-click Pressure Acoustics, Frequency Domain (acpr) and choose Node Group.
- 2 In the Settings window for Group, type Homogenized Model in the Label text field.

The outer boundaries need a radiation condition to make sure that no waves are reflected at this interface.

Cylindrical Wave Radiation I

- I Right-click Homogenized Model and choose Radiation Conditions> Cylindrical Wave Radiation.
- 2 Select Boundary 154 only.
- 3 In the Settings window for Cylindrical Wave Radiation, locate the Cylindrical Wave Radiation section.
- **4** Specify the \mathbf{r}_0 vector as

x1	x
y1	y

Homogenized Model

Now, proceed to add a **Background Pressure Field** for each of the geometries. The field is a plane wave propagating in the x direction. The phase of this field is updated on each of the domains to make sure that the incident acoustic field is the same in all the domains.

Background Pressure Field I

- I In the Physics toolbar, click **Domains** and choose Background Pressure Field.
- 2 In the Settings window for Background Pressure Field, locate the Domain Selection section.
- 3 From the Selection list, choose Selection: Homogenized Cloak Domain.
- **4** Locate the **Background Pressure Field** section. In the p_0 text field, type 1.
- **5** From the c list, choose **From material**.
- 6 From the Material list, choose Air (mat1).
- 7 In the ϕ text field, type acpr.bpf1.k*x1.

Symmetry I

- I In the Physics toolbar, click Boundaries and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- 3 From the Selection list, choose Selection: Symmetry Boundaries, Homogenized Cloak.

Anisotropic Acoustics 1

- I In the Physics toolbar, click **Domains** and choose **Anisotropic Acoustics**.
- **2** Select Domain 4 only.
- 3 In the Settings window for Anisotropic Acoustics, locate the Coordinate System Selection section.
- 4 From the Coordinate system list, choose Cylindrical System 2 (sys2).

No Cloak Model

- I In the Model Builder window, right-click Pressure Acoustics, Frequency Domain (acpr) and choose **Node Group**.
- 2 In the Settings window for Group, type No Cloak Model in the Label text field.

Cylindrical Wave Radiation 2

- I In the Physics toolbar, click Boundaries and choose Cylindrical Wave Radiation.
- 2 Select Boundary 209 only.
- 3 In the Settings window for Cylindrical Wave Radiation, locate the Cylindrical Wave Radiation section.

4 Specify the \mathbf{r}_0 vector as

x2	x
у2	у

Background Pressure Field 2

- I In the Physics toolbar, click **Domains** and choose **Background Pressure Field**.
- 2 In the Settings window for Background Pressure Field, locate the Domain Selection section.
- 3 From the Selection list, choose Selection: No Cloak Domain.
- **4** Locate the **Background Pressure Field** section. In the p_0 text field, type 1.
- **5** From the c list, choose From material.
- **6** In the ϕ text field, type acpr.bpf2.k*x2.

Symmetry 2

- I In the Physics toolbar, click Boundaries and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- 3 From the Selection list, choose Selection: Symmetry Boundaries, No Cloak.

50 Laver Cloak Model

- I Right-click Pressure Acoustics, Frequency Domain (acpr) and choose Node Group.
- 2 In the Settings window for Group, type 50 Layer Cloak Model in the Label text field.

Cylindrical Wave Radiation 3

- I In the Physics toolbar, click Boundaries and choose Cylindrical Wave Radiation.
- 2 Select Boundary 153 only.
- 3 In the Settings window for Cylindrical Wave Radiation, locate the Cylindrical Wave Radiation section.
- **4** Specify the \mathbf{r}_0 vector as

хЗ	x
уЗ	у

Background Pressure Field 3

- I In the Physics toolbar, click **Domains** and choose **Background Pressure Field**.
- 2 In the Settings window for Background Pressure Field, locate the Domain Selection section.

- 3 From the Selection list, choose Selection: 50 Layer Cloak Domain.
- **4** Locate the **Background Pressure Field** section. In the p_0 text field, type 1.
- **5** From the c list, choose **From material**.
- **6** In the ϕ text field, type acpr.bpf3.k*x3.

Symmetry 3

- I In the Physics toolbar, click Boundaries and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- 3 From the Selection list, choose Selection: Symmetry Boundaries, 50 Layer Cloak.

20 Layer Cloak Model

- I Right-click Pressure Acoustics, Frequency Domain (acpr) and choose Node Group.
- 2 In the Settings window for Group, type 20 Layer Cloak Model in the Label text field.

Cylindrical Wave Radiation 4

- I In the Physics toolbar, click Boundaries and choose Cylindrical Wave Radiation.
- **2** Select Boundary 208 only.
- 3 In the Settings window for Cylindrical Wave Radiation, locate the Cylindrical Wave Radiation section.
- **4** Specify the \mathbf{r}_0 vector as

x4	x
y4	у

Background Pressure Field 4

- I In the Physics toolbar, click **Domains** and choose Background Pressure Field.
- 2 In the Settings window for Background Pressure Field, locate the Domain Selection section.
- 3 From the Selection list, choose Selection: 20 Layer Cloak Domain.
- **4** Locate the **Background Pressure Field** section. In the p_0 text field, type 1.
- **5** From the c list, choose **From material**.
- 6 In the o text field, type acpr.bpf4.k*x4.

Symmetry 4

- I In the Physics toolbar, click Boundaries and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- 3 From the Selection list, choose Selection: Symmetry Boundaries, 20 Layer Cloak.

MESH I

In this model, the mesh is set up manually. Proceed by directly adding the desired mesh component.

Free Triangular 1

In the Mesh toolbar, click Free Triangular.

Size

- I In the Model Builder window, click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 Click the **Custom** button.
- 4 Locate the Element Size Parameters section. In the Maximum element size text field, type lam0/6.

In general, five to six second-order elements per wavelength are needed to resolve the waves. For more details, see Meshing (Resolving the Waves) in the Acoustics Module User's Guide. This model uses six elements per wavelength.

5 Click | Build Selected.

We will add a mapped mesh in the acoustic cloak domains. As the domains have properties that change with the radius, it is a good idea to keep a structured mesh with elements along the varying direction.

Mapped I

- I In the Mesh toolbar, click Mapped.
- 2 In the Settings window for Mapped, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- 4 From the Selection list, choose Selection: Acoustic Cloak.

Since the variation of properties is more pronounced toward the inner center of the acoustic cloak, add a skewed distribution to make sure that the mesh is sufficiently fine throughout the domain.

Distribution 1

- I Right-click Mapped I and choose Distribution.
- 2 Select Boundary 4 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 From the Distribution type list, choose Predefined.
- 5 In the Number of elements text field, type 20.

6 In the Element ratio text field, type 4.

The same distribution should be applied to the other edge of the domain, but with the reverse direction.

Distribution 2

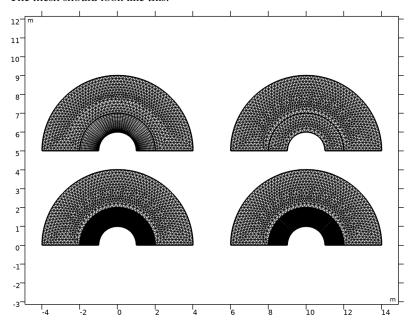
- I In the Model Builder window, right-click Mapped I and choose Distribution.
- 2 Select Boundary 55 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 From the Distribution type list, choose Predefined.
- 5 In the Number of elements text field, type 20.
- **6** In the **Element ratio** text field, type **4**.
- 7 Select the Reverse direction check box.

Distribution 3

- I Right-click Mapped I and choose Distribution.
- **2** Select Boundary 156 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 In the Number of elements text field, type 40.

5 In the Model Builder window, right-click Mesh I and choose Build All.

The mesh should look like this.



STUDY I

Step 1: Frequency Domain

- I In the Model Builder window, under Study I click Step I: Frequency Domain.
- 2 In the Settings window for Frequency Domain, locate the Study Settings section.
- 3 In the Frequencies text field, type f0.
- 4 In the Home toolbar, click **Compute**.

The following steps will update the default plots to include a cleaner view of the model with annotations of the different cloaks.

RESULTS

Total Acoustic Pressure (acpr)

- I In the Settings window for 2D Plot Group, type Total Acoustic Pressure (acpr) in the Label text field.
- 2 Locate the Plot Settings section. Clear the Plot dataset edges check box.

Line 1

- I Right-click Total Acoustic Pressure (acpr) and choose Line.
- 2 In the Settings window for Line, locate the Expression section.
- **3** In the **Expression** text field, type **0**.
- 4 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- 6 From the Color list, choose Black.

Selection I

- I Right-click Line I and choose Selection.
- **2** Select Boundaries 1, 2, 105–108, 151–156, and 208–211 only.

Total Acoustic Pressure (acpr)

In the Model Builder window, under Results click Total Acoustic Pressure (acpr).

Table Annotation I

- I In the Total Acoustic Pressure (acpr) toolbar, click More Plots and choose **Table Annotation.**
- 2 In the Settings window for Table Annotation, locate the Data section.
- 3 From the Source list, choose Local table.
- **4** In the table, enter the following settings:

x-coordinate	y-coordinate	Annotation
x1	y1+4.3[m]	Homogenized cloak
x2	y2+4.3[m]	No cloak
x3	y3+4.3[m]	50 Layer cloak
x4	y4+4.3[m]	20 Layer cloak

- 5 Locate the Coloring and Style section. Clear the Show point check box.
- **6** From the **Anchor point** list, choose **Center**.
- 7 In the Total Acoustic Pressure (acpr) toolbar, click **1** Plot.
- **8** Click the **Zoom Extents** button in the **Graphics** toolbar.

Total Acoustic Pressure (acpr)

I In the Model Builder window, click Total Acoustic Pressure (acpr).

2 In the Total Acoustic Pressure (acpr) toolbar, click Plot.

The plot should look like that in Figure 2.

Sound Pressure Level (actr)

I In the Model Builder window, under Results right-click Sound Pressure Level (acpr) and choose Delete.

Remove the default Sound Pressure Level plot and duplicate the Total Acoustic Pressure plot created previously. Doing this, you make sure that the annotations will be carried over to the new plots.

Total Sound Pressure Level (acpr)

- I In the Model Builder window, right-click Total Acoustic Pressure (acpr) and choose
- 2 In the Settings window for 2D Plot Group, type Total Sound Pressure Level (acpr) in the Label text field.

Surface I

- I In the Model Builder window, expand the Total Sound Pressure Level (acpr) node, then click Surface 1.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type acpr.Lp t.
- 4 Locate the Coloring and Style section. Click Change Color Table.
- 5 In the Color Table dialog box, select Rainbow>Rainbow in the tree.
- 6 Click OK.
- 7 In the Settings window for Surface, locate the Coloring and Style section.
- 8 From the Scale list, choose Linear.
- 9 In the Total Sound Pressure Level (acpr) toolbar, click on Plot.

The plot should look like that in Figure 3.

Scattered Sound Pressure Level (acpr)

- I In the Model Builder window, right-click Total Sound Pressure Level (acpr) and choose Duplicate.
- 2 In the Settings window for 2D Plot Group, type Scattered Sound Pressure Level (acpr) in the Label text field.

Surface 1

I In the Model Builder window, expand the Scattered Sound Pressure Level (acpr) node, then click Surface 1.

- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type acpr.Lp s.

Selection 1

- I Right-click Surface I and choose Selection.
- 2 Select Domains 1, 2, 54, and 55 only.
- 3 In the Scattered Sound Pressure Level (acpr) toolbar, click Plot. The plot should look like that in Figure 4.

Effective Speed of Sound in Principal Directions

- I In the Home toolbar, click **Add Plot Group** and choose **2D Plot Group**.
- 2 In the Settings window for 2D Plot Group, type Effective Speed of Sound in Principal Directions in the Label text field.
- 3 Click to expand the Selection section. From the Geometric entity level list, choose Domain.
- 4 Select Domain 4 only.
- 5 Locate the Plot Settings section. From the View list, choose New view to create a dedicated view for this plot.
- 6 Locate the Selection section. Select the Apply to dataset edges check box.
- 7 Click the Zoom Extents button in the Graphics toolbar.
- **9** Click to expand the **Title** section. From the **Title type** list, choose **Label**.

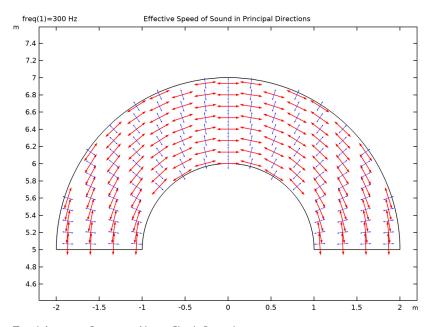
Principal Stress Surface 1

- I In the Effective Speed of Sound in Principal Directions toolbar, click More Plots and choose Principal Stress Surface.
- 2 In the Settings window for Principal Stress Surface, locate the Principal Components section.
- 3 Find the Principal values subsection. In row Value, set Second to 0.
- 4 In row Value, set Third to 0.5.
- 5 Find the Principal directions subsection. In row X, set First to acpr.c eff1x+ acpr.c_eff2x.
- 6 In row Y, set First to acpr.c eff1y+acpr.c eff2y.
- 7 In row Y, set Second to 0.
- 8 In row X, set Third to acpr.c eff3x.

- 9 In row Y, set Third to acpr.c_eff3y.
- **IO** In row **Z**, set **Third** to **0**.

The first and second effective speeds of sound are identical in magnitude, but one of them is perpendicular to the plane. Both terms are added to the first direction so that the arrows are consistent.

II In the Effective Speed of Sound in Principal Directions toolbar, click **Plot**. The plot should look like this.



Total Acoustic Pressure Along Cloak Boundary

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Total Acoustic Pressure Along Cloak Boundary in the Label text field.
- 3 Click to expand the **Title** section. From the **Title type** list, choose **Label**.
- 4 Locate the Plot Settings section.
- 5 Select the y-axis label check box. In the associated text field, type Pressure (Pa).

Line Graph 1

- I Right-click Total Acoustic Pressure Along Cloak Boundary and choose Line Graph.
- **2** Select Boundary 156 only.

- 3 In the Settings window for Line Graph, locate the y-Axis Data section.
- 4 In the Expression text field, type acpr.p_b.
- 5 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 6 Click to expand the **Legends** section. Select the **Show legends** check box.
- 7 From the Legends list, choose Manual.
- **8** In the table, enter the following settings:

Legends Background pressure field

- 9 Click to expand the Coloring and Style section. Find the Line markers subsection. From the Marker list, choose Circle.
- 10 From the Positioning list, choose Interpolated.
- II In the Number text field, type 100.

Line Grabh 2

- I Right-click Line Graph I and choose Duplicate.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type acpr.p t.
- 4 Locate the Coloring and Style section. Find the Line markers subsection. From the Marker list, choose None.
- **5** Locate the **Legends** section. In the table, enter the following settings:

Legends Homogenized cloak model

Line Graph 3

- I Right-click Line Graph 2 and choose Duplicate.
- 2 In the Settings window for Line Graph, locate the Selection section.
- 3 Click Clear Selection.
- 4 Select Boundary 155 only.
- **5** Locate the **Legends** section. In the table, enter the following settings:

Legends 50 layer cloak model

Line Graph 4

- I Right-click Line Graph 3 and choose Duplicate.
- 2 In the Settings window for Line Graph, locate the Selection section.
- 3 Click Clear Selection.
- 4 Select Boundary 210 only.
- **5** Locate the **Legends** section. In the table, enter the following settings:

Legends 20 layer cloak model

6 In the Total Acoustic Pressure Along Cloak Boundary toolbar, click Plot.

The plot should look like that in Figure 5.

Speed of Sound in Homogenized Material

- I In the Home toolbar, click Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Speed of Sound in Homogenized Material in the Label text field.
- **3** Locate the **Title** section. From the **Title type** list, choose **Label**.
- 4 Locate the Plot Settings section.
- 5 Select the y-axis label check box. In the associated text field, type Speed of sound (m/ s).

Line Grabh I

- I Right-click Speed of Sound in Homogenized Material and choose Line Graph.
- 2 Select Boundary 55 only.
- 3 In the Settings window for Line Graph, locate the y-Axis Data section.
- 4 In the Expression text field, type acpr. c eff1.
- 5 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **6** In the **Expression** text field, type x.
- 7 Locate the **Legends** section. Select the **Show legends** check box.
- 8 Find the Include subsection. Clear the Solution check box.
- **9** Select the **Description** check box.

Line Graph 2

- I Right-click Line Graph I and choose Duplicate.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.

- 3 In the Expression text field, type acpr.c_eff3.
- 4 Click the y-Axis Log Scale button in the Graphics toolbar.
- 5 In the Speed of Sound in Homogenized Material toolbar, click 💿 Plot. The plot should look like that in Figure 6.

Appendix: Geometry Sequence Instructions

ADD COMPONENT

In the **Home** toolbar, click **Add Component** and choose **2D**.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file acoustic cloaking geom sequence parameters.txt.

GEOMETRY I

Circle I (c1)

- I In the Geometry toolbar, click Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type 4.
- 4 In the Sector angle text field, type 180.
- **5** Locate the **Position** section. In the **x** text field, type x1.
- 6 In the y text field, type y1.
- 7 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	R2
Layer 2	R1

Circle 2 (c2)

I In the Geometry toolbar, click (Circle.

- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type 2*R2.
- **4** In the **Sector angle** text field, type 180.
- **5** Locate the **Position** section. In the **x** text field, type x2.
- 6 In the y text field, type y2.
- 7 Locate the Layers section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	R2
Layer 2	R1

Circle 3 (c3)

- I In the Geometry toolbar, click Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type 2*R2.
- 4 In the Sector angle text field, type 180.
- **5** Locate the **Position** section. In the **x** text field, type **x3**.
- 6 In the y text field, type y3.
- 7 Locate the Layers section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	R2
Layer 2	(R2-R1)/50
Layer 3	(R2-R1)/50
Layer 4	(R2-R1)/50
Layer 5	(R2-R1)/50
Layer 6	(R2-R1)/50
Layer 7	(R2-R1)/50
Layer 8	(R2-R1)/50
Layer 9	(R2-R1)/50
Layer 10	(R2-R1)/50
Layer 11	(R2-R1)/50
Layer 12	(R2-R1)/50
Layer 13	(R2-R1)/50
Layer 14	(R2-R1)/50

Layer name	Thickness (m)
Layer 15	(R2-R1)/50
Layer 16	(R2-R1)/50
Layer 17	(R2-R1)/50
Layer 18	(R2-R1)/50
Layer 19	(R2-R1)/50
Layer 20	(R2-R1)/50
Layer 21	(R2-R1)/50
Layer 22	(R2-R1)/50
Layer 23	(R2-R1)/50
Layer 24	(R2-R1)/50
Layer 25	(R2-R1)/50
Layer 26	(R2-R1)/50
Layer 27	(R2-R1)/50
Layer 28	(R2-R1)/50
Layer 29	(R2-R1)/50
Layer 30	(R2-R1)/50
Layer 31	(R2-R1)/50
Layer 32	(R2-R1)/50
Layer 33	(R2-R1)/50
Layer 34	(R2-R1)/50
Layer 35	(R2-R1)/50
Layer 36	(R2-R1)/50
Layer 37	(R2-R1)/50
Layer 38	(R2-R1)/50
Layer 39	(R2-R1)/50
Layer 40	(R2-R1)/50
Layer 41	(R2-R1)/50
Layer 42	(R2-R1)/50
Layer 43	(R2-R1)/50
Layer 44	(R2-R1)/50
Layer 45	(R2-R1)/50
Layer 46	(R2-R1)/50
Layer 47	(R2-R1)/50
Layer 48	(R2-R1)/50

Layer name	Thickness (m)
Layer 49	(R2-R1)/50
Layer 50	(R2-R1)/50
Layer 51	(R2-R1)/50

Circle 4 (c4)

- I In the Geometry toolbar, click Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type 2*R2.
- 4 In the Sector angle text field, type 180.
- **5** Locate the **Position** section. In the **x** text field, type **x4**.
- 6 In the y text field, type y4.
- 7 Locate the Layers section. In the table, enter the following settings:

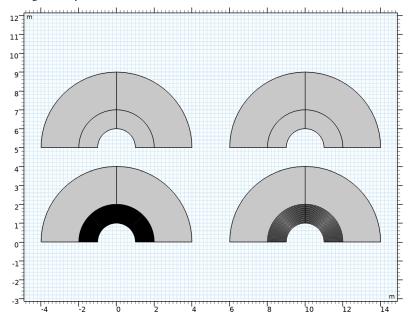
Layer name	Thickness (m)
Layer 1	R2
Layer 2	(R2-R1)/20
Layer 3	(R2-R1)/20
Layer 4	(R2-R1)/20
Layer 5	(R2-R1)/20
Layer 6	(R2-R1)/20
Layer 7	(R2-R1)/20
Layer 8	(R2-R1)/20
Layer 9	(R2-R1)/20
Layer 10	(R2-R1)/20
Layer 11	(R2-R1)/20
Layer 12	(R2-R1)/20
Layer 13	(R2-R1)/20
Layer 14	(R2-R1)/20
Layer 15	(R2-R1)/20
Layer 16	(R2-R1)/20
Layer 17	(R2-R1)/20
Layer 18	(R2-R1)/20
Layer 19	(R2-R1)/20
Layer 20	(R2-R1)/20
Layer 21	(R2-R1)/20

- 8 Click Build All Objects.
- **9** Click the **Toom Extents** button in the **Graphics** toolbar.

Delete Entities I (del1)

- I In the Model Builder window, right-click Geometry I and choose Delete Entities.
- 2 In the Settings window for Delete Entities, locate the Entities or Objects to Delete section.
- 3 From the Geometric entity level list, choose Domain.
- 4 On the object c1, select Domain 3 only.
- 5 On the object c2, select Domain 3 only.
- 6 On the object c3, select Domain 52 only.
- 7 On the object c4, select Domain 22 only.
- 8 Click Pauld Selected.

The geometry should look like this.



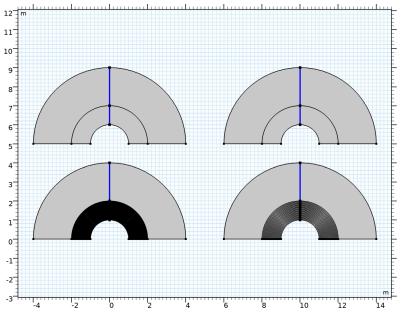
Now that the geometry has been generated, you can proceed to simplify some of its features.

Ignore Edges I (ige I)

- I In the Geometry toolbar, click \times Virtual Operations and choose Ignore Edges.
- 2 In the Settings window for Ignore Edges, locate the Input section.

- 3 Clear the Ignore adjacent vertices check box.
- 4 Click Paste Selection.
- 5 In the Paste Selection dialog box, type 54-106 183-205 in the Selection text field.
- 6 Click OK.

The selection should look like this.



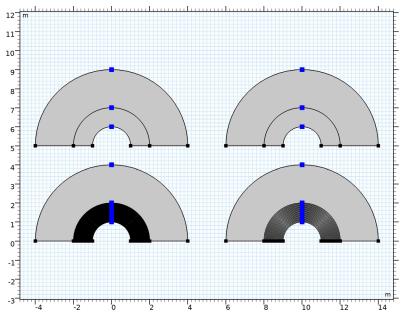
7 In the Settings window for Ignore Edges, click 📳 Build Selected.

Ignore Vertices I (igvI)

- I In the Geometry toolbar, click 🇠 Virtual Operations and choose Ignore Vertices.
- 2 In the Settings window for Ignore Vertices, locate the Input section.
- 3 Click Paste Selection.
- 4 In the Paste Selection dialog box, type 56-110 191-215 in the Selection text field.

5 Click OK.

The selection should look like this.



6 In the Settings window for Ignore Vertices, click | Build Selected.

With this clean geometry, you can proceed to generate a few selections that will be used in the model.

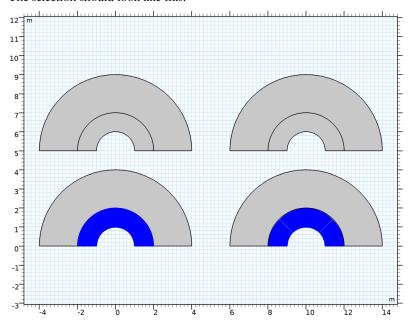
GEOMETRY I

Selection: Material I

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Explicit Selection.
- 2 In the Settings window for Explicit Selection, type Selection: Material 1 in the Label text field.
- 3 Locate the Entities to Select section. Click Paste Selection.
- 4 In the Paste Selection dialog box, type igv1: 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76 in the Selection text field.

5 Click OK.

The selection should look like this.

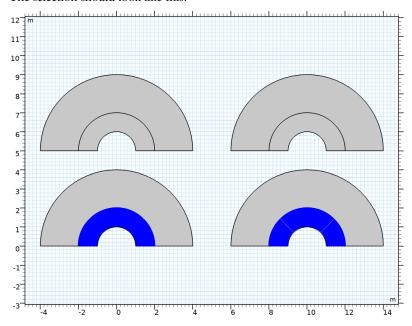


Selection: Material 2

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Explicit Selection.
- 2 In the Settings window for Explicit Selection, type Selection: Material 2 in the Label text field.
- 3 Locate the Entities to Select section. Click Paste Selection.
- 4 In the Paste Selection dialog box, type igv1: 3, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 56, 59, 61, 63, 65, 67, 69, 71, 73, 75 in the Selection text field.

5 Click OK.

The selection should look like this.



Selection: Homogenized Cloak

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Explicit Selection.
- 2 In the Settings window for Explicit Selection, type Selection: Homogenized Cloak in the Label text field.
- 3 Locate the Entities to Select section. Click Paste Selection.
- 4 In the Paste Selection dialog box, type igv1: 4 in the Selection text field.
- 5 Click OK.
- 6 In the Geometry toolbar, click **Build All**. It is also possible to select entities based on their spatial position. Through the next steps you will add a few selections based on the position.

Selection: 50 Layer Cloak

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.
- 2 In the Settings window for Box Selection, type Selection: 50 Layer Cloak in the Label text field.
- 3 Locate the Box Limits section. In the x minimum text field, type -2.5+x3.

- 4 In the x maximum text field, type 2.5+x3.
- 5 In the y minimum text field, type 0+y3.
- 6 In the y maximum text field, type 2.5+y3.
- 7 Locate the Output Entities section. From the Include entity if list, choose Entity inside box.
- 8 Click Pauld Selected.

Selection: 20 Layer Cloak

- I In the Geometry toolbar, click **Selections** and choose **Box Selection**.
- 2 In the Settings window for Box Selection, type Selection: 20 Layer Cloak in the Label text field.
- 3 Locate the Box Limits section. In the x minimum text field, type -2.5+x4.
- 4 In the x maximum text field, type 2.5+x4.
- 5 In the y minimum text field, type 0+y4.
- 6 In the y maximum text field, type 2.5+y4.
- 7 Locate the Output Entities section. From the Include entity if list, choose Entity inside box.
- 8 Click **P** Build Selected.

Selection: Acoustic Cloak

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Union Selection.
- 2 In the Settings window for Union Selection, type Selection: Acoustic Cloak in the **Label** text field.
- 3 Locate the **Input Entities** section. Click Add.
- 4 In the Add dialog box, in the Selections to add list, choose Selection: Homogenized Cloak, Selection: 50 Layer Cloak, and Selection: 20 Layer Cloak.
- 5 Click OK.

Selection: Symmetry Boundaries, Homogenized Cloak

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.
- 2 In the Settings window for Box Selection, type Selection: Symmetry Boundaries, Homogenized Cloak in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Boundary.
- 4 Locate the Box Limits section. In the x minimum text field, type -5+x1.
- 5 In the x maximum text field, type 5+x1.

- 6 In the y minimum text field, type -0.5+y1.
- 7 In the y maximum text field, type 0.5+y1.
- 8 Locate the Output Entities section. From the Include entity if list, choose Entity inside box.

Selection: Symmetry Boundaries, No Cloak

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.
- 2 In the Settings window for Box Selection, type Selection: Symmetry Boundaries, No Cloak in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Boundary.
- 4 Locate the Box Limits section. In the x minimum text field, type -5+x2.
- 5 In the x maximum text field, type 5+x2.
- 6 In the y minimum text field, type -0.5+y2.
- 7 In the y maximum text field, type 0.5+y2.
- 8 Locate the Output Entities section. From the Include entity if list, choose Entity inside box.

Selection: Symmetry Boundaries, 50 Layer Cloak

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.
- 2 In the Settings window for Box Selection, type Selection: Symmetry Boundaries, 50 Layer Cloak in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Boundary.
- 4 Locate the Box Limits section. In the x minimum text field, type -5+x3.
- 5 In the x maximum text field, type 5+x3.
- 6 In the y minimum text field, type -0.5+y3.
- 7 In the y maximum text field, type 0.5+y3.
- 8 Locate the Output Entities section. From the Include entity if list, choose Entity inside box.

Selection: Symmetry Boundaries, 20 Layer Cloak

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.
- 2 In the Settings window for Box Selection, type Selection: Symmetry Boundaries, 20 Layer Cloak in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Boundary.
- 4 Locate the **Box Limits** section. In the **x minimum** text field, type -5+x4.

- 5 In the x maximum text field, type 5+x4.
- 6 In the y minimum text field, type -0.5+y4.
- 7 In the y maximum text field, type 0.5+y4.
- 8 Locate the Output Entities section. From the Include entity if list, choose Entity inside box.

Selection: Homogenized Cloak Domain

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.
- 2 In the Settings window for Box Selection, type Selection: Homogenized Cloak Domain in the Label text field.
- 3 Locate the Box Limits section. In the x minimum text field, type -5+x1.
- 4 In the x maximum text field, type 5+x1.
- 5 In the y minimum text field, type -0.5+y1.
- 6 In the y maximum text field, type 0.5+y1.

Selection: No Cloak Domain

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.
- 2 In the Settings window for Box Selection, type Selection: No Cloak Domain in the Label text field.
- 3 Locate the Box Limits section. In the x minimum text field, type -5+x2.
- 4 In the x maximum text field, type 5+x2.
- 5 In the y minimum text field, type -0.5+y2.
- 6 In the y maximum text field, type 0.5+y2.

Selection: 50 Layer Cloak Domain

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.
- 2 In the Settings window for Box Selection, type Selection: 50 Layer Cloak Domain in the Label text field.
- 3 Locate the Box Limits section. In the x minimum text field, type -5+x3.
- 4 In the x maximum text field, type 5+x3.
- 5 In the y minimum text field, type -0.5+y3.
- 6 In the y maximum text field, type 0.5+y3.

Selection: 20 Layer Cloak Domain

I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.

- 2 In the Settings window for Box Selection, type Selection: 20 Layer Cloak Domain in the Label text field.
- 3 Locate the Box Limits section. In the x minimum text field, type -5+x4.
- 4 In the x maximum text field, type 5+x4.
- **5** In the **y minimum** text field, type -0.5+y4.
- 6 In the y maximum text field, type 0.5+y4.
- 7 In the Geometry toolbar, click **Build All**.