Created in COMSOL Multiphysics 6.1

Superconducting Wire

Introduction

Current can flow in a superconducting wire with practically zero resistance, although factors including temperature, current density, and magnetic field can limit this phenomenon. This application solves a time-dependent problem of a current building up in a superconducting wire close to the critical current density. This application is based on a suggestion by Dr. Roberto Brambilla, CESI, Superconductivity Dept., Milano, Italy.

The Dutch physicist Heike Kamerlingh Onnes discovered superconductivity in 1911. He cooled mercury to the temperature of liquid helium (4 K) and observed that its resistivity suddenly disappeared. Research in superconductivity reached a peak during the 1980s in terms of activity and discoveries, especially when scientists uncovered the superconductivity of ceramics. In particular, it was during this decade that researchers discovered YBCO — a ceramic superconductor composed of yttrium, barium, copper, and oxygen with a critical temperature above the temperature of liquid nitrogen. However, researchers have not yet created a room-temperature superconductor, so much work remains for the broad commercialization of this area.

This application illustrates how current builds up in a cross section of a superconducting wire; it also shows where critical currents produce a swelling in the non-superconducting region.

Model Definition

The dependence of resistivity on the amount of current makes it difficult to solve the problem using the Magnetic Fields interface. The reason is that a circular dependency arises because the current-density calculation contains the resistivity, leading to a resistivity that is dependent on itself.

An alternative approach uses the magnetic field as the dependent variable, and you can then calculate the current as

$$\mathbf{J} = \nabla \times \mathbf{H}$$

The electric field is a function of the current, and Faraday's law determines the complete system as in

$$\nabla \times \mathbf{E}(\mathbf{J}) = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

where $\mathbf{E}(\mathbf{J})$ is the current-dependent electric field. The model calculates this field with the empirical formula

$$\mathbf{E}(\mathbf{J}) = \begin{cases} \mathbf{0} & |\mathbf{J}| < J_C \\ E_0\left(\dfrac{|\mathbf{J}| - J_C}{J_C}\right)^{\alpha}\dfrac{\mathbf{J}}{|\mathbf{J}|} & |\mathbf{J}| \geq J_C \end{cases}$$

where E_0 and α are constants determining the nonlinear behavior of the transition to zero resistivity, and J_C is the critical current density, which decreases as temperature increases.

For the superconductor YBCO, this model uses the following parameter values (Ref. 1):

PARAMETER	VALUE
E_0	0.0836168 V/m
α	1.449621256
J_C	17 MA
T_C	92 K

Systems with two curl operators are best dealt with using vector elements (edge elements). This is the default element for the physics interfaces in the AC/DC Module that solve similar equations. This particular formulation for the superconducting system is available in the AC/DC Module as the Magnetic Field Formulation interface.

For symmetry reasons, the current density has only a z-component.

The model controls current through the wire with its outer boundary condition. Because Ampère's law must hold around the wire, a line integral around it must add up to the current through the wire. Cylindrical symmetry results in a known magnetic field at the outer boundary

$$\oint \mathbf{H} \cdot \mathbf{dl} = 2\pi r H_{\phi} = I_{\mathrm{wire}} \Rightarrow H_{\phi} = \frac{I_{\mathrm{wire}}}{2\pi r}$$

This is applied as a constraint on the tangential component of the vector field.

Results and Discussion

The model applies a simple transient exponential function as the current through the wire, reaching a final value of 1 MA. This extremely large current is necessary if the superconducting wire is to reach its critical current density. Plotting the current density at different time instants shows the swelling of the region in which the current flows. This swelling comes from the transition out of the superconducting state at current densities exceeding J_C. Figure 1 presents a plot of the current density at t = 0.1 s.

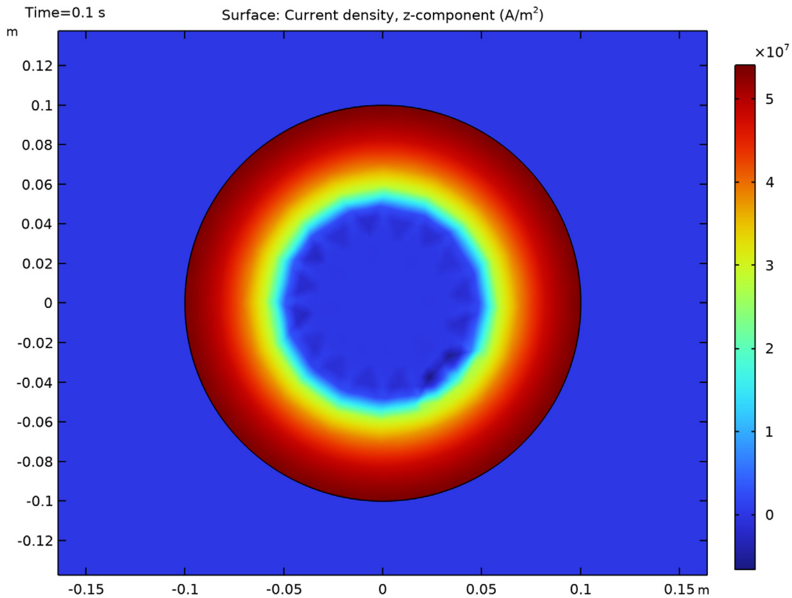


Figure 1: The current density at 0.1 s.

Reference

1. R. Pecher, M.D. McCulloch, S.J. Chapman, L. Prigozhin, and C.M. Elliotth, “3D-modelling of bulk type-II superconductors using unconstrained H-formulation,” *6th European Conf. Applied Superconductivity*, EUCAS, 2003.

Application Library path: `ACDC_Module/Devices,_Superconducting/superconducting_wire`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.
2 In the **Select Physics** tree, select **AC/DC>Electromagnetic Fields>Vector Formulations>Magnetic Field Formulation (mfh)**.
3 Click **Add**.
4 Click **Study**.
5 In the **Select Study** tree, select **General Studies>Time Dependent**.
6 Click **Done**.

GEOMETRY 1

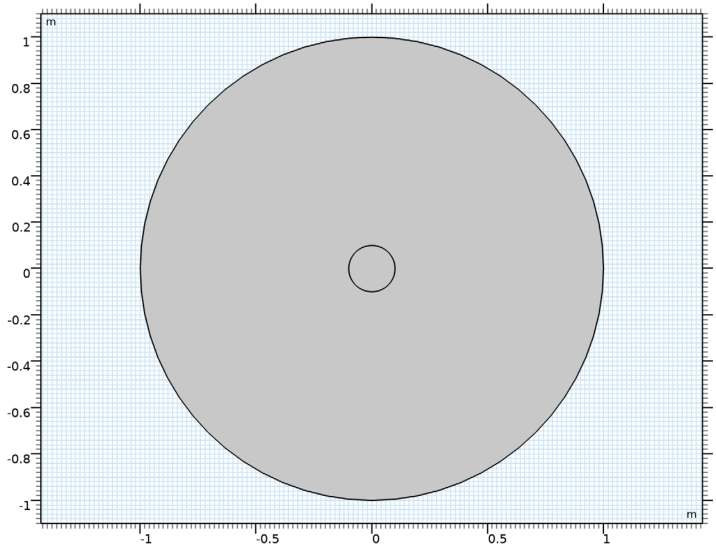
Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.
2 Keep all the default values.

Circle 2 (c2)

1 In the **Geometry** toolbar, click **Circle**.
2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.
3 In the **Radius** text field, type `0.1`.

4 Click **Build All Objects**.



GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `superconducting_wire_parameters.txt`.

DEFINITIONS

Define a step function that will be used in the expression of the superconductor characteristic.

Step 1 (step1)

In the **Home** toolbar, click f(X) **Functions** and choose **Local>Step**.

Variables 1

1 In the **Home** toolbar, click a= **Variables** and choose **Local Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
I1	I0*(1-exp(-t/tau))	A	
H0phi	I1/(2*pi*sqrt(x^2+y^2))	A/m	

In order to simplify the application of the boundary condition, add a cylindrical coordinate system.

Cylindrical System 2 (sys2)

In the **Definitions** toolbar, click **Coordinate Systems** and choose **Cylindrical System**.

MAGNETIC FIELD FORMULATION (MFH)

1 Click the **Show More Options** button in the **Model Builder** toolbar.

2 In the **Show More Options** dialog box, in the tree, select the check box for the node **Physics>Advanced Physics Options**.

3 Click **OK**.

When using the **Magnetic Field Formulation** physics with superconducting material, it is necessary to turn off the automatic divergence constraint, that can lead to instability.

4 In the **Model Builder** window, under **Component 1 (comp1)** click **Magnetic Field Formulation (mfh)**.

5 In the **Settings** window for **Magnetic Field Formulation**, click to expand the **Divergence Constraint** section.

6 Clear the **Activate divergence constraint** check box.

Faraday's Law 1

Set the constitutive relation for the default Faraday's Law node to use **Resistivity**.

1 In the **Model Builder** window, under **Component 1 (comp1)> Magnetic Field Formulation (mfh)** click **Faraday's Law 1**.

2 In the **Settings** window for **Faraday's Law**, locate the **Constitutive Relation Jc-E** section.

3 From the **Conduction model** list, choose **Electrical resistivity**.

Faraday's Law 2

1 In the **Physics** toolbar, click **Domains** and choose **Faraday's Law**.

2 Select Domain 2 only.

3 In the **Settings** window for **Faraday's Law**, locate the **Constitutive Relation Jc-E** section.

4 From the **Conduction model** list, choose **E-J characteristic**.

The Faraday's Law feature will use the material data specified in the Superconductor material.

Set up the boundary condition for the magnetic field.

Magnetic Field 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Magnetic Field**.

2 Select Boundaries 1, 2, 5, and 8 only.

3 In the **Settings** window for **Magnetic Field**, locate the **Coordinate System Selection** section.

4 From the **Coordinate system** list, choose **Cylindrical System 2 (sys2)**.

5 Locate the **Magnetic Field** section. Specify the $\mathbf{H}_0$ vector as

`0`	r
`H0phi`	phi
`0`	a

MATERIALS

Add the materials used in the model. For the domain surrounding the wire, create a material representing Air.

Air

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Air` in the **Label** text field.

The **Magnetic Field Formulation** physics requires a finite resistivity in all domains.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic

Property	Variable	Value	Unit	Property group
Resistivity	res_iso ; resii = res_iso, resij = 0	`rho_air`	Ω·m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

For the superconductor, create a custom material that uses the ’E-J Characteristic’ model.

Superconductor

1 Right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Superconductor` in the **Label** text field.

3 Select Domain 2 only.

Fill in the relative permittivity and permeability.

4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

Now add a subnode that provides the material model for the superconductor.

5 Locate the **Material Properties** section. In the **Material properties** tree, select **Electromagnetic Models>E-J Characteristic**.

6 Click **Add to Material**.

7 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Property group
Electric field norm	normE	`E0*(((normJ-Jc)/Jc)*step1((normJ-Jc)/1[A/m^2]))^alpha`	E-J characteristic

Mesh 1

Proceed with creating the mesh. Use a finer mesh in the superconducting domain to resolve the current density.

Free Triangular 1

In the **Mesh** toolbar, click **Free Triangular**.

Size 1

1 Right-click **Free Triangular 1** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domain 2 only.

5 Locate the **Element Size** section. Click the **Custom** button.

6 Locate the **Element Size Parameters** section.

7 Select the **Maximum element size** check box. In the associated text field, type `0.02`.

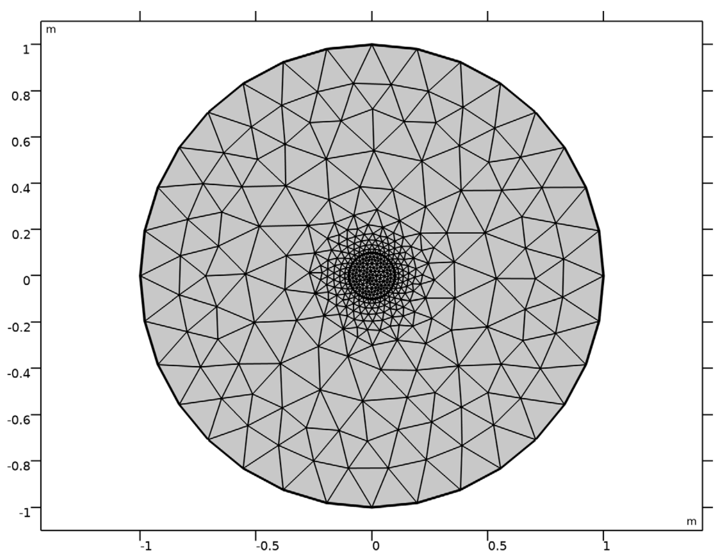
Size

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 From the **Predefined** list, choose **Coarse**.

4 Click **Build All**.



STUDY 1

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 Click **Range**.

4 In the **Range** dialog box, type `0.005` in the **Step** text field.

5 In the **Stop** text field, type `0.1`.

6 Click **Replace**.

To improve the accuracy of the time-dependent solution, specify a small initial time step and a maximum step size.

Solution 1 (sol1)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Time-Dependent Solver 1**.

3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.

4 Select the **Initial step** check box. In the associated text field, type `1e-9`.

5 From the **Maximum step constraint** list, choose **Constant**.

6 In the **Maximum step** text field, type `1e-3`.

7 In the **Model Builder** window, expand the **Study 1>Solver Configurations> Solution 1 (sol1)>Time-Dependent Solver 1** node, then click **Fully Coupled 1**.

8 In the **Settings** window for **Fully Coupled**, click to expand the **Method and Termination** section.

9 From the **Jacobian update** list, choose **Once per time step**.

10 In the **Study** toolbar, click **Compute**.

RESULTS

Magnetic Flux Density Norm (mfh)

The default plot, shown after the computation is completed, visualizes the norm of the magnetic flux density.

Create a plot group to visualize the current density.

Current Density

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Current Density` in the **Label** text field.

Surface 1

1 Right-click **Current Density** and choose **Surface**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Magnetic Field Formulation>Currents and charge>Current density - A/m²>mfh.Jz - Current density, z-component**.

3 In the **Current Density** toolbar, click **Plot**.

4 Click the **Zoom In** button on the **Graphics toolbar** two or three times to get a closer view of the wire.

Under the **Export** node, it is possible to create an animation of the evolution of the current density distribution.

Animation 1

1 In the **Results** toolbar, click **Animation** and choose **Player**.

2 In the **Settings** window for **Animation**, locate the **Scene** section.

3 From the **Subject** list, choose **Current Density**.

4 Click the **Play** button in the **Graphics** toolbar.

The animation can also be saved to file by selecting **File** from the **Target** list box then clicking **Export**.