Created in COMSOL Multiphysics 6.1

Magnetic Field from an Infinite Conductor

Introduction

This introduction model creates a simple model of the magnetostatics problem with a wire of infinite length carrying current, which is commonly found in textbooks. Since there is an analytical solution to this problem, the model can be used to compare theory with numerical results from the simulation. The wire still has a finite radius in this example, giving different solutions for the regions inside and outside of the wire.

Model Definition

First, consider the analytical solution to this problem, which can be used for comparison. Outside an infinite wire of radius r_i — that is, for $r > r_i$ — the magnetic flux density, **B**, satisfies

$$\mathbf{B} = \frac{\mu_0 I_0}{2\pi r}\hat{\varphi}$$

where μ_0 is the permeability of vacuum, I_0 is the current through the wire, and $\hat{\varphi}$ is a unit vector in the azimuthal direction. Inside the wire, where $r < r_i$, the solution instead reads

$$\mathbf{B} = \frac{\mu_0 J_0 r}{2}\hat{\varphi}$$

where J_0 is the current density in the wire. These expressions can, for example, be derived by using the integral formulation of Ampère's law. From the symmetry of the problem it follows that the magnetic-field is orientated purely in the azimuthal direction and that its strength depends only on the radial distance from the wire.

The geometry of the infinite wire will be created in two different ways:

- By using a 2D component, where the Magnetic Fields interface solves for the out-of-plane magnetic vector potential. The 2D component then corresponds to the cross section of the infinite wire.
- By using a 2D axisymmetric component and solving for the in-plane magnetic vector potential, which when revolved becomes the full wire. The Magnetic Insulation boundary condition, which applies the constraint $\mathbf{n} \times \mathbf{A} = 0$ on the horizontal boundaries, causes this to be equivalent to the infinite wire as well.

When it comes to the numerical solvers, there is one major difference between the two cases: Solving only for the out-of-plane magnetic vector potential component acts as an inherent gauge fix for the problem by removing an extra degree of freedom. In contrast, the second way involves solving for two in-plane components, implying that the problem is ungauged. For this reason, it cannot be solved with a direct solver, which requires a unique solution in order to converge. One way of addressing this issue is to add a Gauge Fixing for A-Field feature to that component. However, as an interesting exercise, a different approach will be illustrated here, namely changing the default direct solver to an iterative one. Iterative solvers can solve even ungauged problems, finding one solution out of the many possible ones. It is of course important to remember that all measurable quantities, like the magnetic flux density, are the same no matter which potential is chosen.

Results

The magnetic flux density norm in the 2D component is plotted in Figure 1, while the magnetic flux density norm in the 2D axisymmetric component is shown in Figure 2 (symmetry cross section) and Figure 3 (revolved geometry). Finally, Figure 4 shows a plot of the magnetic flux density as a function of the radial coordinate for the two numerical solutions as well as for the analytical solution discussed above. There, it can be seen how well the numerical solutions corresponding to the two different approaches agree with the analytical solution.

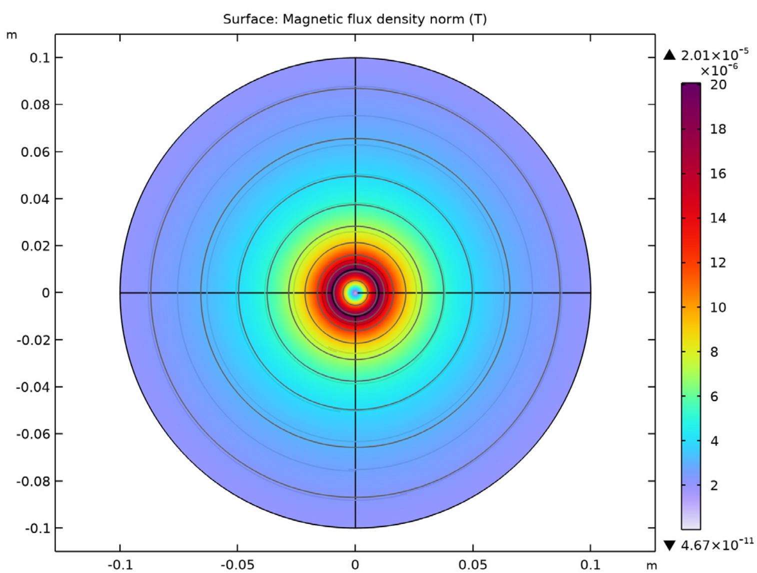

Figure 1: Magnetic flux density norm in the 2D component.

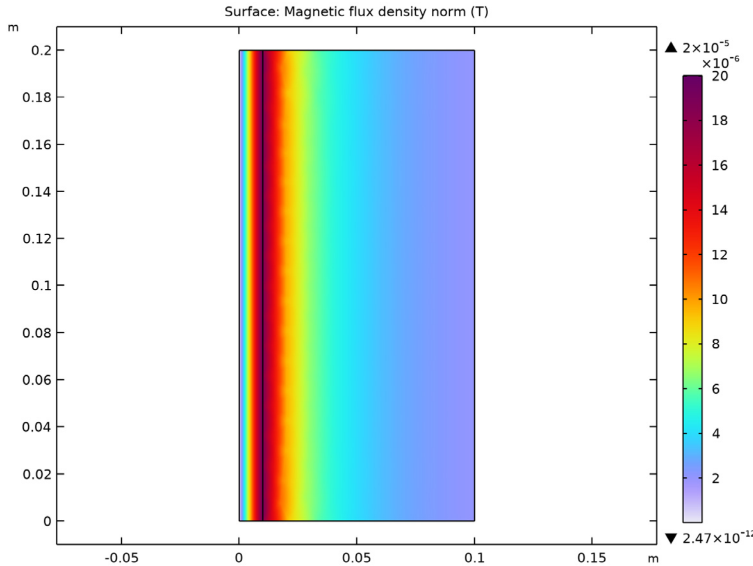

Figure 2: Magnetic flux density norm in the 2D axisymmetric component.

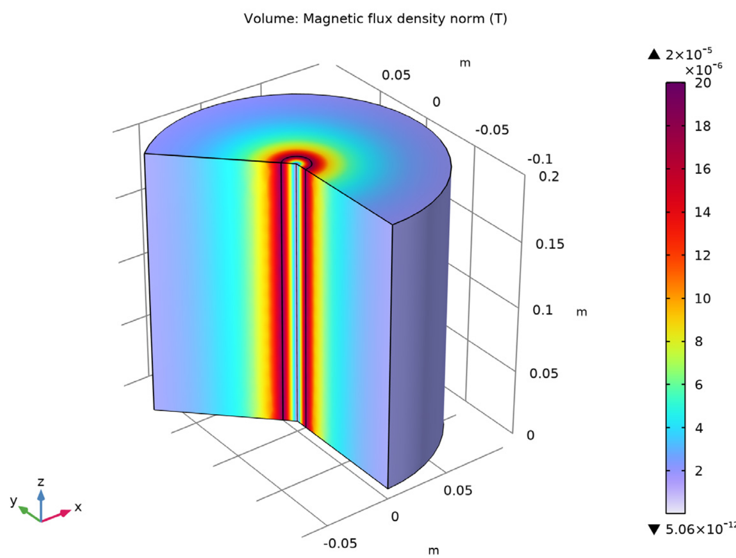

Figure 3: Magnetic flux density norm in the 2D axisymmetric component, shown in the full revolved geometry.

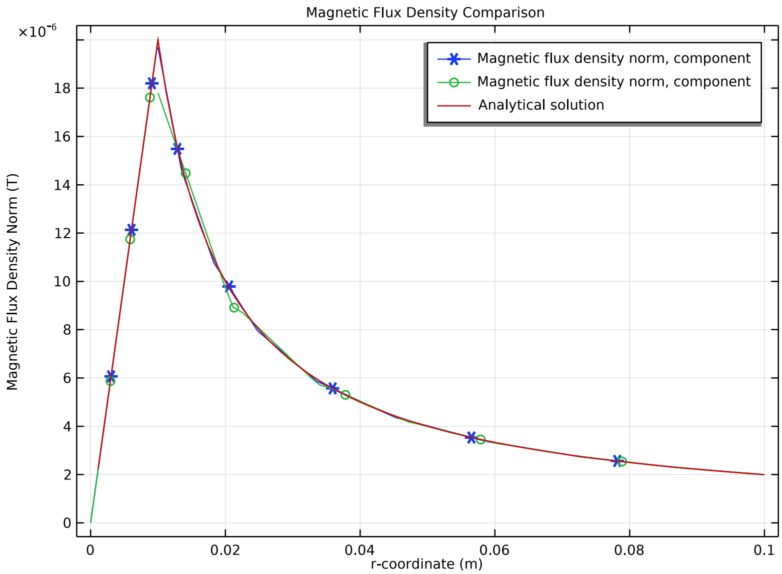

Figure 4: The magnetic flux density norm plotted as a function of the radial coordinate. Here, the numerical solutions from the two different components are shown, along with the analytical solution.

Application Library path: `ACDC_Module/Introductory_Magnetostatics/magnetic_field_infinite_conductor`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **AC/DC>Electromagnetic Fields>Magnetic Fields (mf)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

First, define some parameters that will be used when building the model.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
ri	1[cm]	0.01 m	Radius of the conductor
ro	10[cm]	0.1 m	Radius of the computation domain
I0	1[A]	1 A	Conducting current
J0	I0/(pi*ri^2)	3183.1 A/m²	Current density in the conductor

GEOMETRY 1

One way of constructing the geometry of the infinite wire is to use a 2D component. Since the Magnetic Fields physics interface by default only solves for the out-of-plane component of the magnetic vector potential, this is equivalent to having a wire of infinite length, where the 2D geometry corresponds to the cross section.

Circle 1 (c1)

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Geometry 1** node.

2 Right-click **Geometry 1** and choose **Circle**.

3 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

4 In the **Radius** text field, type `ro`.

5 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	ro-ri

Point 1 (pt1)

In the **Geometry** toolbar, click **Point**.

Line Segment 1 (ls1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Line Segment**.

2 On the object **pt1**, select Point 1 only.

3 In the **Settings** window for **Line Segment**, locate the **Endpoint** section.

4 Find the **End vertex** subsection. Click to select the **Activate Selection** toggle button.

5 On the object **c1**, select Point 7 only.

6 In the **Geometry** toolbar, click **Build All**.

MATERIALS

Air

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Air` in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

Conductor

1 Right-click **Air** and choose **Duplicate**.

2 In the **Settings** window for **Material**, type `Conductor` in the **Label** text field.

3 Select Domain 4 only.

4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	1e6	S/m	Basic

The only modification that needs to be done for the physics, is adding a current density to the wire.

MAGNETIC FIELDS (MF)

External Current Density 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Magnetic Fields (mf)** and choose **External Current Density**.

2 Select Domain 4 only.

3 In the **Settings** window for **External Current Density**, locate the **External Current Density** section.

4 Specify the $\mathbf{J}_e$ vector as

0	x
0	y
J0	z

STUDY 1

In the **Home** toolbar, click **Compute**.

Now, add a second, 2D axisymmetric, component that will be used to illustrate a different approach to modeling the same problem.

ADD COMPONENT

In the **Model Builder** window, right-click the root node and choose **Add Component> 2D Axisymmetric**.

GEOMETRY 2

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `ro`.

4 In the **Height** text field, type `2*ro`.

5 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	ri

6 Select the **Layers to the left** check box.

7 Clear the **Layers on bottom** check box.

8 In the **Geometry** toolbar, click **Build All**.

ADD PHYSICS

1 In the **Home** toolbar, click **Add Physics** to open the **Add Physics** window.

2 Go to the **Add Physics** window.

3 In the tree, select **AC/DC>Electromagnetic Fields>Magnetic Fields (mf)**.

4 Click **Add to Component 2** in the window toolbar.

MATERIALS

Air

1 In the **Model Builder** window, under **Component 2 (comp2)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Air` in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic

Property	Variable	Value	Unit	Property group
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

Conductor

1 Right-click **Air** and choose **Duplicate**.

2 In the **Settings** window for **Material**, type `Conductor` in the **Label** text field.

3 Select Domain 1 only.

4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	1e6	S/m	Basic

MAGNETIC FIELDS 2 (MF2)

1 In the **Model Builder** window, under **Component 2 (comp2)** click **Magnetic Fields 2 (mf2)**.

2 In the **Settings** window for **Magnetic Fields**, locate the **Components** section.

3 From the **Field components solved for** list, choose **In-plane vector potential**.

External Current Density 1

1 In the **Physics** toolbar, click **Domains** and choose **External Current Density**.

2 Select Domain 1 only.

3 In the **Settings** window for **External Current Density**, locate the **External Current Density** section.

4 Specify the $\mathbf{J}_e$ vector as

0	r
0	phi
J0	z

Now, add a another stationary study for the second component, and make sure that the two studies only solve for their respective physics interfaces.

STUDY 1

Step 1: Stationary

1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 In the table, enter the following settings:

Physics interface	Solve for	Equation form
Magnetic Fields (mf)	√	Automatic (Stationary)
Magnetic Fields 2 (mf2)		Automatic (Stationary)

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

STUDY 2

Step 1: Stationary

1 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

2 In the table, enter the following settings:

Physics interface	Solve for	Equation form
Magnetic Fields (mf)		Automatic (Stationary)
Magnetic Fields 2 (mf2)	√	Automatic (Stationary)

Due to the ungauged formulation, the suggested default solver will be an iterative one.

3 In the **Home** toolbar, click **Compute**.

Now, it is time to compare the results with the known analytical solutions. Add plots of the magnetic flux density computed from each component, as well as a plot containing the known analytical solution to the problem. Note that the solution is different in the regions inside and outside of the wire.

RESULTS

Magnetic Flux Density Comparison

1. In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2. In the **Settings** window for **1D Plot Group**, type `Magnetic Flux Density Comparison` in the **Label** text field.
3. Click to expand the **Title** section. From the **Title type** list, choose **Label**.
4. Locate the **Data** section. From the **Dataset** list, choose **None**.

Line Graph 1

1. Right-click **Magnetic Flux Density Comparison** and choose **Line Graph**.
2. In the **Settings** window for **Line Graph**, locate the **Data** section.
3. From the **Dataset** list, choose **Study 1/Solution 1 (sol1)**.
4. Select Boundaries 3 and 5 only.
5. Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
6. In the **Expression** text field, type `x`.
7. Click to expand the **Legends** section. Select the **Show legends** check box.
8. Find the **Include** subsection. Clear the **Solution** check box.
9. Select the **Description** check box.
10. Find the **Prefix and suffix** subsection. In the **Suffix** text field, type `, component 1`.
11. Click to expand the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.
12. From the **Positioning** list, choose **Interpolated**.

Magnetic Flux Density Comparison

1. In the **Model Builder** window, click **Magnetic Flux Density Comparison**.
2. In the **Settings** window for **1D Plot Group**, locate the **Plot Settings** section.
3. Select the **x-axis label** check box. In the associated text field, type `r-coordinate (m)`.
4. Select the **y-axis label** check box. In the associated text field, type `Magnetic Flux Density Norm (T)`.

Line Graph 2

1. Right-click **Magnetic Flux Density Comparison** and choose **Line Graph**.
2. In the **Settings** window for **Line Graph**, locate the **Data** section.
3. From the **Dataset** list, choose **Study 2/Solution 2 (3) (sol2)**.
4. Select Boundaries 2 and 5 only.

5. Locate the **y-Axis Data** section. In the **Expression** text field, type `mf2.normB`.
6. Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
7. In the **Expression** text field, type `r`.
8. Locate the **Legends** section. Select the **Show legends** check box.
9. Find the **Include** subsection. Clear the **Solution** check box.
10. Select the **Description** check box.
11. Find the **Prefix and suffix** subsection. In the **Suffix** text field, type `, component 2`.
12. Locate the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.
13. From the **Positioning** list, choose **Interpolated**.

Line Graph 3

1. Right-click **Magnetic Flux Density Comparison** and choose **Line Graph**.
2. In the **Settings** window for **Line Graph**, locate the **Data** section.
3. From the **Dataset** list, choose **Study 2/Solution 2 (3) (sol2)**.
4. Select Boundaries 2 and 5 only.
5. Locate the **y-Axis Data** section. In the **Expression** text field, type `mu0_const*J0/2*r*(r<=ri)+mu0_const*I0/(2*pi*r)*(r>ri)`.
6. Locate the **Legends** section. Select the **Show legends** check box.
7. Find the **Include** subsection. Clear the **Solution** check box.
8. Select the **Expression** check box.
9. From the **Legends** list, choose **Manual**.
10. In the table, enter the following settings:

Legends
Analytical solution

11. In the **Magnetic Flux Density Comparison** toolbar, click  **Plot**.