

Magnetic Field of a Helmholtz Coil

A Helmholtz coil is a parallel pair of identical circular coils spaced one radius apart and wound so that the current flows through both coils in the same direction. This winding results in a uniform magnetic field between the coils with the primary component parallel to the axis of the two coils. The uniform field is the result of the sum of the two field components parallel to the axis of the coils and the difference between the components perpendicular to the same axis.

The purpose of the device is to allow scientists and engineers to perform experiments and tests that require a known ambient magnetic field. Helmholtz field generation can be static, time varying DC, or AC, depending on application.

Applications include canceling Earth's magnetic field for certain experiments; generating magnetic fields for determining magnetic shielding effectiveness or susceptibility of electronic equipment to magnetic fields; calibration of magnetometers and navigational equipment; and biomagnetic studies.

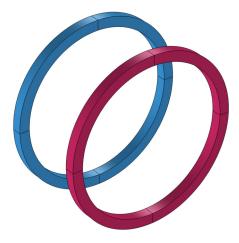


Figure 1: The Helmholtz coil consists of two coaxial circular coils, one radius apart along the axial direction. The coils carry parallel currents of equal magnitude.

The application shows how to compute the magnetic field with two different approaches, one using the Magnetic Fields interface and the other the Magnetic Fields, Currents Only interface. The model geometry is shown in Figure 2.

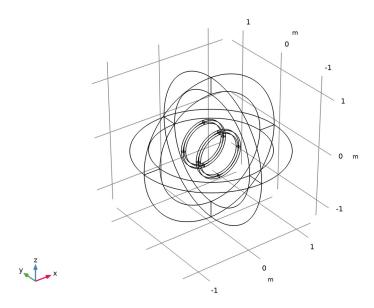


Figure 2: The model geometry.

DOMAIN EQUATIONS

Assuming static currents and fields, the magnetic vector potential **A** must satisfy the following equation:

$$\nabla \times (\boldsymbol{\mu}^{-1} \nabla \times \mathbf{A}) = \mathbf{J}_{\mathbf{A}}$$

where μ is the permeability, and \boldsymbol{J}_{e} denotes the externally applied current density.

The relations between the magnetic field \mathbf{H} , the magnetic flux density \mathbf{B} and the potential are given by

$$\mathbf{B} = \nabla \times \mathbf{A}$$
$$\mathbf{H} = \mathbf{u}^{-1} \mathbf{B}$$

This model uses the permeability of vacuum, that is, $\mu \approx 4\pi \times 10^{-7}$ H/m. The external current density is computed using a homogenized model for the coils, each one made by 10 wire turns and excited by a current of 0.25 mA. The currents are specified to be parallel for the two coils.

SPATIAL DERIVATIVE OF MAGNETIC FIELD

Computing the spatial derivative of the magnetic field or magnetic flux density is useful in areas such as radiology, magnetophoresis, particle accelerators, and geophysics. One of the most important use cases is the design of magnetic resonance imaging (MRI) machines, where it is necessary to analyze not only the field strength but also the spatial variation of the field. This application demonstrates how to compute the spatial derivative of the magnetic flux density in the postprocessing step.

Results and Discussion

Figure 3 shows the magnetic flux density between the coils. The flux is relatively uniform in the region between the coils. This uniformity is the main property and often the sought feature of a Helmholtz coil.

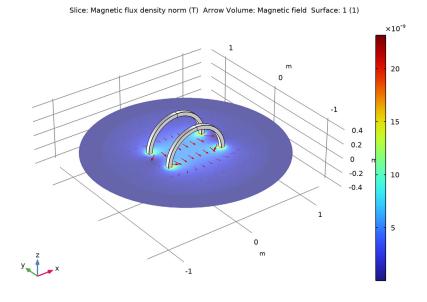


Figure 3: The slice plot shows the magnetic flux density norm. The arrows indicate the magnetic field (H) strength and direction.

Figure 4 and Figure 5 compare the results from using the two different physics interfaces.

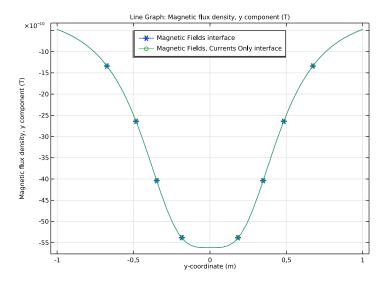


Figure 4: Comparison of the y component of the B field along the centerline of the Helmholtz coil using two different approaches.

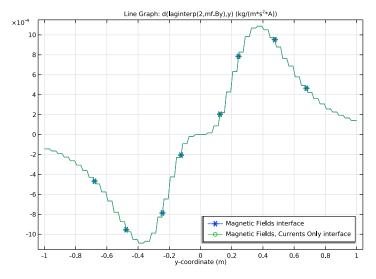


Figure 5: Comparison of the gradient (with respect to the y direction) of the y component of the B field along the centerline of the Helmholtz coil.

Application Library path: ACDC_Module/Devices,_Inductive/helmholtz_coil

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select AC/DC>Electromagnetic Fields>Magnetic Fields (mf).
- 3 Click Add.
- 4 In the Select Physics tree, select AC/DC>Electromagnetic Fields>Vector Formulations> Magnetic Fields, Currents Only (mfco).
- 5 Click Add.
- 6 Click 🔵 Study.
- 7 In the Select Study tree, select General Studies>Stationary.
- 8 Click M Done.

GLOBAL DEFINITIONS

Parameters I

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
10	0.25[mA]	2.5E-4 A	Coil current

GEOMETRY I

Work Plane I (wpl)

I In the Geometry toolbar, click Work Plane.

2 In the Settings window for Work Plane, click Show Work Plane.

Work Plane I (wp I)>Plane Geometry

In the Model Builder window, click Plane Geometry.

Work Plane I (wpl)>Square I (sql)

- I In the Work Plane toolbar, click Square.
- 2 In the Settings window for Square, locate the Size section.
- 3 In the Side length text field, type 0.05.
- 4 Locate the **Position** section. From the **Base** list, choose **Center**.
- 5 In the xw text field, type -0.4.
- 6 In the yw text field, type 0.2.

Work Plane I (wb I)>Square 2 (sq2)

- I In the Work Plane toolbar, click Square.
- 2 In the Settings window for Square, locate the Size section.
- 3 In the Side length text field, type 0.05.
- 4 Locate the Position section. From the Base list, choose Center.
- 5 In the xw text field, type -0.4.
- 6 In the yw text field, type -0.2.

Revolve I (rev1)

In the Model Builder window, under Component I (compl)>Geometry I right-click Work Plane I (wpl) and choose Revolve.

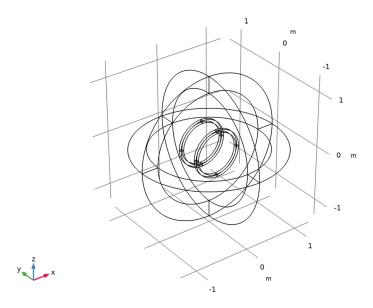
Sphere I (sph I)

- I In the Geometry toolbar, click \bigcirc Sphere.
- 2 In the Settings window for Sphere, locate the Size section.
- 3 In the Radius text field, type 1.3.
- 4 Click to expand the Layers section. In the table, enter the following settings:

Layer name	Thickness (m)		
Layer 1	0.3		

- 5 Click **Build All Objects**.
- **6** Click the **Zoom Extents** button in the **Graphics** toolbar.

7 The geometry is now complete. To see its interior, click the **Wireframe Rendering** button in the **Graphics** toolbar.



Line Segment I (Is I)

- I In the Geometry toolbar, click More Primitives and choose Line Segment.
- 2 On the object sph1, select Point 4 only.
- 3 In the Settings window for Line Segment, locate the Endpoint section.
- **4** Find the **End vertex** subsection. Click to select the **Activate Selection** toggle button.
- 5 On the object sph1, select Point 9 only.

DEFINITIONS

Next, define the Infinite Element Domain.

Infinite Element Domain I (ie I)

- I In the Definitions toolbar, click on Infinite Element Domain.
- 2 In the Settings window for Infinite Element Domain, locate the Geometry section.
- 3 From the Type list, choose Spherical.
- 4 Select Domains 1-4 and 8-11 only.

DEFINITIONS

Hide for Physics 1

- I In the Model Builder window, right-click View I and choose Hide for Physics.
- 2 In the Settings window for Hide for Physics, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- **4** Select Boundaries 6 and 10 only.

MATERIALS

Define the materials for the model.

ADD MATERIAL

- I In the Home toolbar, click **Add Material** to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click 4 Add Material to close the Add Material window. By default, the first material added is applied on all domains.

Add another material for the coil domains.

MATERIALS

Coil Insulator

- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Coil Insulator in the Label text field.
- **3** Select Domains 6 and 7 only.

4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso; murii = mur_iso, murij = 0	1	I	Basic
Electrical conductivity	sigma_iso; sigmaii = sigma_iso, sigmaij = 0	6e7[S/m]	S/m	Basic
Relative permittivity	epsilonr_iso; epsilonrii = epsilonr_iso, epsilonrij = 0	1	I	Basic

MAGNETIC FIELDS (MF)

Coil I

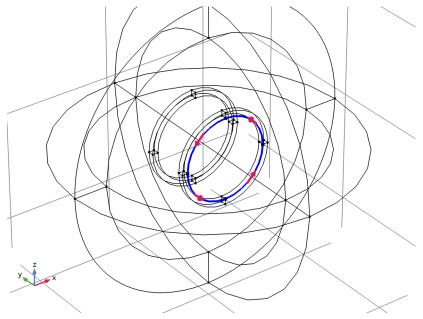
- I In the Model Builder window, under Component I (compl) right-click Magnetic Fields (mf) and choose the domain setting Coil.
- 2 Select Domain 6 only.
- 3 In the Settings window for Coil, locate the Coil section.
- 4 From the Conductor model list, choose Homogenized multiturn.
- 5 From the Coil type list, choose Circular.
- **6** In the I_{coil} text field, type I0.

In order to specify the direction of the wires in the circular coil, use the Coil Geometry subfeature to select a group of edges forming a circle. The path of the wires will be automatically computed from the geometry of the selected edges. For the best results, the radius of the circular edges selected should be close to the average radius of the coil.

Coil Geometry 1

- I In the Model Builder window, click Coil Geometry I.
- 2 In the Settings window for Coil Geometry, locate the Edge Selection section.
- 3 Click Clear Selection.

4 Select Edges 25, 26, 46, and 49 only.



Now set up the second coil in the same way.

Coil 2

- I In the Physics toolbar, click **Domains** and choose Coil.
- **2** Select Domain 7 only.
- 3 In the Settings window for Coil, locate the Coil section.
- 4 From the Conductor model list, choose Homogenized multiturn.
- 5 From the Coil type list, choose Circular.
- **6** In the $I_{\rm coil}$ text field, type IO.

Coil Geometry 1

- I In the Model Builder window, click Coil Geometry I.
- 2 In the Settings window for Coil Geometry, locate the Edge Selection section.
- 3 Click Clear Selection.
- 4 Select Edges 30, 31, 72, and 75 only.

MAGNETIC FIELDS, CURRENTS ONLY (MFCO)

In the Model Builder window, under Component I (compl) click Magnetic Fields, Currents Only (mfco).

Conductor I

- I In the Physics toolbar, click **Domains** and choose Conductor.
- 2 Select Domain 6 only.

Terminal I

- I In the Model Builder window, expand the Conductor I node, then click Terminal I.
- **2** Select Boundary 13 only.
- 3 In the Settings window for Terminal, locate the Terminal section.
- **4** In the I_0 text field, type 10*I0.

Conductor 2

- I In the Physics toolbar, click **Domains** and choose Conductor.
- **2** Select Domain 7 only.

Terminal I

- I In the Model Builder window, expand the Conductor 2 node, then click Terminal I.
- 2 Select Boundary 20 only.
- 3 In the Settings window for Terminal, locate the Terminal section.
- **4** In the I_0 text field, type 10*I0.

MESH I

Edge I

- I In the Mesh toolbar, click A Boundary and choose Edge.
- 2 Select Edge 40 only.

Distribution 1

- I Right-click **Edge I** and choose **Distribution**.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 In the Number of elements text field, type 50.

Free Tetrahedral I

- I In the Mesh toolbar, click A Free Tetrahedral.
- 2 In the Settings window for Free Tetrahedral, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.

4 Select Domains 5–7 only.

Size 1

- I Right-click Free Tetrahedral I and choose Size.
- 2 Select Domains 6 and 7 only.
- 3 In the Settings window for Size, locate the Element Size section.
- 4 Click the **Custom** button.
- 5 Locate the Element Size Parameters section.
- 6 Select the Maximum element size check box. In the associated text field, type 0.05.

Swebt I

In the Mesh toolbar, click A Swept.

Distribution I

- I Right-click Swept I and choose Distribution.
- 2 Right-click Distribution I and choose Build All.

STUDY I

Stationary 2

In the Study toolbar, click Study Steps and choose Stationary>Stationary.

Step 1: Stationary

- I In the Model Builder window, click Step I: Stationary.
- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check box for Magnetic Fields, Currents Only (mfco).

Step 2: Stationary 2

- I In the Model Builder window, click Step 2: Stationary 2.
- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check box for Magnetic Fields (mf).
- 4 In the Model Builder window, click Study 1.
- 5 In the Settings window for Study, locate the Study Settings section.
- 6 Clear the Generate default plots check box.
- 7 In the Study toolbar, click **Compute**.

Add a selection to the computed dataset to exclude the outer boundaries.

DEFINITIONS

Coils

- I In the **Definitions** toolbar, click **\(\frac{1}{2} \) Explicit**.
- 2 Select Domains 6 and 7 only.
- 3 In the Settings window for Explicit, locate the Output Entities section.
- 4 From the Output entities list, choose Adjacent boundaries.
- 5 In the Label text field, type Coils.

Now add the plots.

RESULTS

In the Model Builder window, expand the Results node.

Study 1/Solution 1 (soll)

In the Model Builder window, expand the Results>Datasets node, then click Study 1/ Solution I (soll).

Selection

- I In the Results toolbar, click hattributes and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 From the Selection list, choose Coils.

Magnetic Flux Density, MF

- I In the Results toolbar, click **3D Plot Group**.
- 2 In the Settings window for 3D Plot Group, type Magnetic Flux Density, MF in the Label text field.

Slice 1

- I Right-click Magnetic Flux Density, MF and choose Slice.
- 2 In the Settings window for Slice, locate the Plane Data section.
- 3 From the Plane list, choose xy-planes.
- 4 In the Planes text field, type 1.
- 5 Click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Magnetic Fields>Magnetic>mf.normB -Magnetic flux density norm - T.
- 6 In the Magnetic Flux Density, MF toolbar, click Plot.

Arrow Volume 1

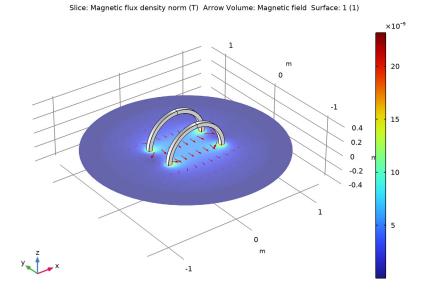
- I In the Model Builder window, right-click Magnetic Flux Density, MF and choose Arrow Volume.
- 2 In the Settings window for Arrow Volume, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)> Magnetic Fields>Magnetic>mf.Hx,mf.Hy,mf.Hz - Magnetic field.
- 3 Locate the Arrow Positioning section. Find the x grid points subsection. In the Points text field, type 24.
- 4 Find the y grid points subsection. In the Points text field, type 10.
- 5 Find the z grid points subsection. In the Points text field, type 1.
- 6 Locate the Coloring and Style section.
- 7 Select the Scale factor check box. In the associated text field, type 25.
- 8 In the Magnetic Flux Density, MF toolbar, click **Plot**.

To make the coil look like a solid object, you can add a surface plot on its boundaries.

Surface I

- I Right-click Magnetic Flux Density, MF and choose Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- **3** In the **Expression** text field, type 1.
- 4 Locate the Coloring and Style section. From the Coloring list, choose Uniform.

5 From the **Color** list, choose **White**.



Next, compare the results of By and Byy calculated from the two interfaces.

Comparison of By

- I In the Home toolbar, click Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Comparison of By in the Label text field.

Line Graph 1

- I Right-click Comparison of By and choose Line Graph.
- 2 Select Edge 40 only.
- 3 In the Settings window for Line Graph, locate the y-Axis Data section.
- 4 In the Expression text field, type mf.By.
- 5 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **6** In the **Expression** text field, type y.
- 7 Click to expand the Coloring and Style section. Find the Line markers subsection. From the Marker list, choose Cycle.
- 8 From the Positioning list, choose Interpolated.
- 9 Click to expand the Legends section. Select the Show legends check box.

10 From the Legends list, choose Manual.

II In the table, enter the following settings:

Legends

Magnetic Fields interface

Line Graph 2

- I Right-click Line Graph I and choose Duplicate.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type mfco.By.
- 4 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- **5** Locate the **Legends** section. In the table, enter the following settings:

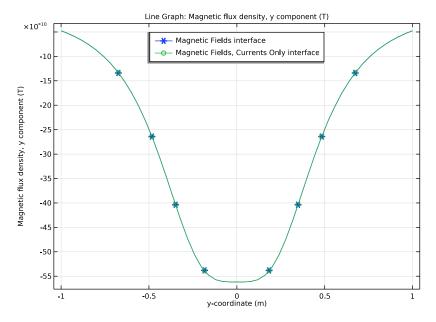
Legends

Magnetic Fields, Currents Only interface

Comparison of By

- I In the Model Builder window, click Comparison of By.
- 2 In the Settings window for ID Plot Group, locate the Legend section.
- **3** From the **Position** list, choose **Upper middle**.

4 In the Comparison of By toolbar, click Plot.



Comparison of Byy

- I Right-click Comparison of By and choose Duplicate.
- 2 In the Model Builder window, click Comparison of By I.
- 3 In the Settings window for ID Plot Group, type Comparison of Byy in the Label text field.
- 4 Locate the Legend section. From the Position list, choose Lower right.

Line Graph 1

- I In the Model Builder window, click Line Graph I.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- 3 In the Expression text field, type d(laginterp(2, mf.By), y).

The mf interface is using Curl shape functions and the higher order spatial derivative is not available in postprocessing. In this case, use the laginterp operator.

Line Graph 2

- I In the Model Builder window, click Line Graph 2.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.

- 3 In the Expression text field, type d(mfco.By,y).
 - The mfco interface is using secondary order Lagrange shape functions and the second derivative is available. The curves of Byy can be improved by using cubic elements.
- 4 In the Comparison of Byy toolbar, click Plot.

