Created in COMSOL Multiphysics 6.1

Shape Optimization of Coils

Introduction

This model demonstrates how to design a coil geometry using gradient-based shape optimization. The objective is to achieve a uniform magnetic field along the coil axis and a field minimum near the axis ends. The model assumes azimuthal symmetry, but one can use the optimization result as inspiration for a 3D design.

Model Definition

The model is set up with 6 coils near the center and one coil at each end with the opposite current direction. The current in the coils at the ends is also optimized to mimic the effect of having coils with partial turns. The result of the shape optimization is shown in Figure 2, while the initial geometry for this model is shown in Figure 1.

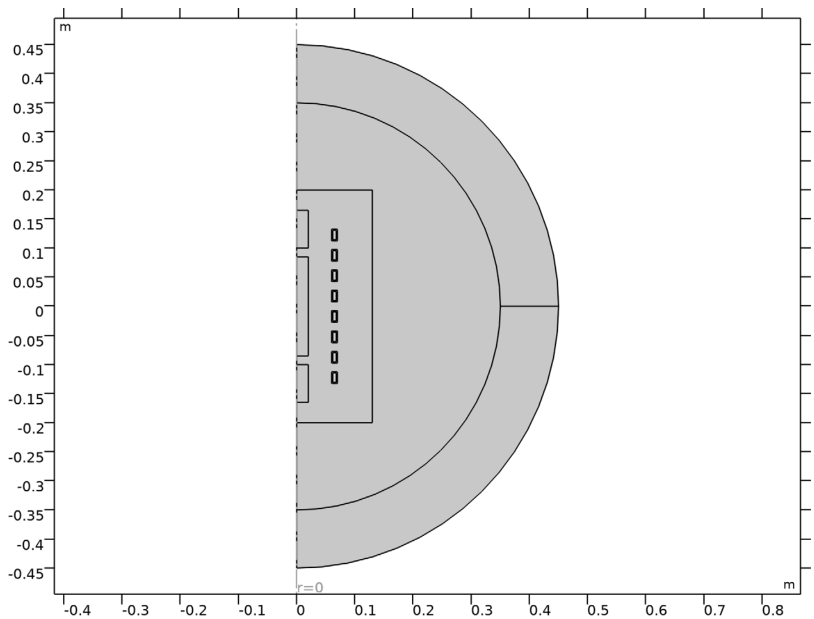


Figure 1: The initial geometry.

The **Transformation** and **Free Shape Domain** features are used to allow the coils to move in the radial direction. There are 8 control variables for the coil positions and one for the current in the outer coils. The IPOPT optimization can solve such problems fast.

The objective function, φ, consists of two terms:

$$\varphi = \varphi_1 + \varphi_2$$

$$\varphi_1 = \int_{\Omega_{\text{inner}}} (|\mathbf{B}| - \mathbf{B}_{\text{avg}})^2 d\Omega \Big/ \int_{\Omega_{\text{inner}}} d\Omega$$

$$\mathbf{B}_{\text{avg}} = \int_{\Omega_{\text{inner}}} |\mathbf{B}| d\Omega \Big/ \int_{\Omega_{\text{inner}}} d\Omega$$

$$\varphi_2 = \int_{\Omega_{\text{outer}}} |\mathbf{B}| d\Omega \Big/ \int_{\Omega_{\text{outer}}} d\Omega$$

Results and Discussion

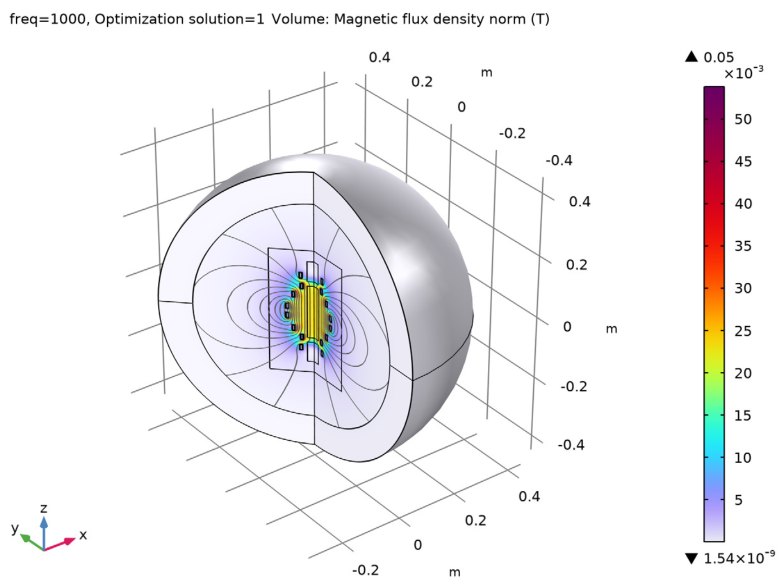


Figure 2: The optimized geometry.

Figure 2 shows the optimized design. The first and second objectives are reduced by around 90% and 50%, respectively. The strength of the magnetic field on the axis is illustrated in Figure 3.

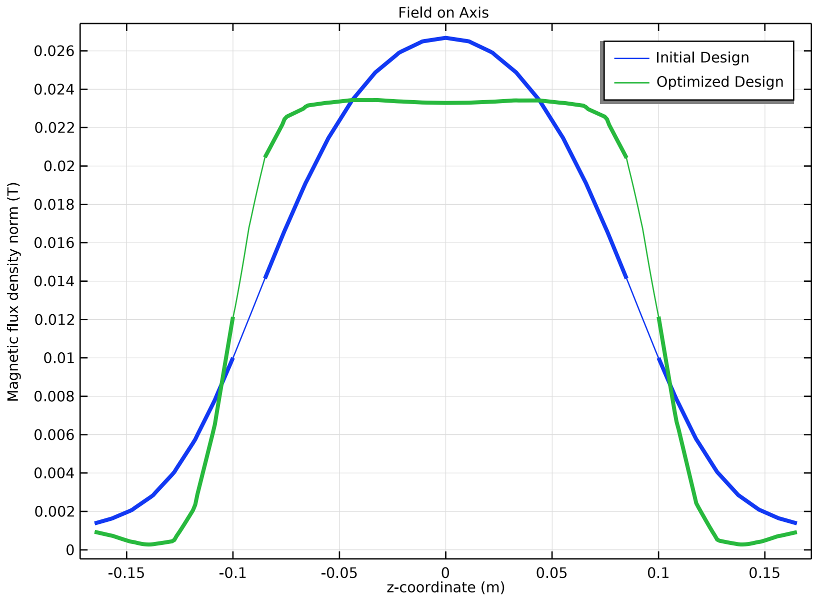


Figure 3: The optimization increases the magnetic field in the middle domain and decreases it in the outer domains.

Notes About the COMSOL Implementation

This model can be constructed in a way that exploits symmetry in the *xy*-plane, leading to a reduction in the computational time.

Application Library path: `ACDC_Module/Electromagnetics_and_Optimization/coil_shape_optimization`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **AC/DC>Electromagnetic Fields>Magnetic Fields (mf)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Frequency Domain**.

6 Click **Done**.

GEOMETRY 1

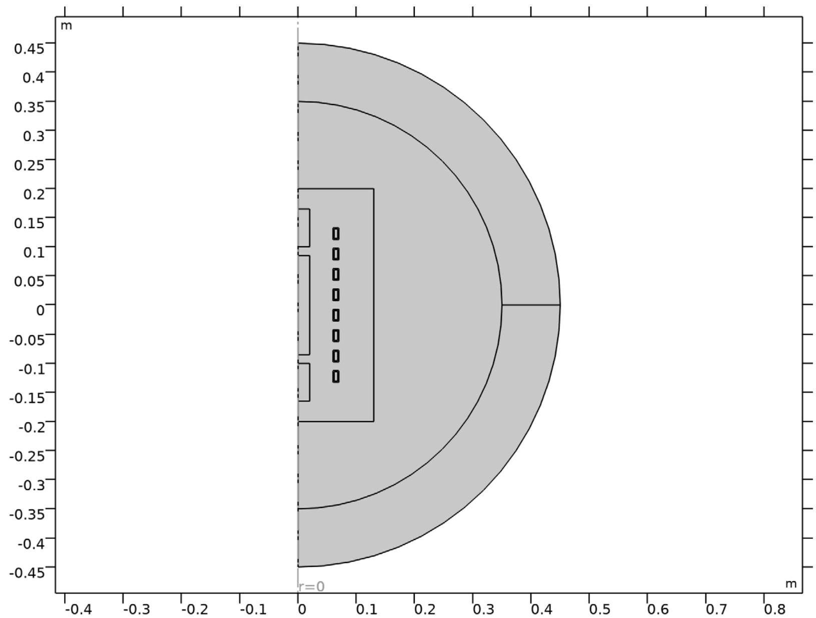
1 In the **Model Builder** window, expand the **Component 1 (comp1)>Geometry 1** node.

2 Right-click **Geometry 1** and choose **Insert Sequence**.

3 Browse to the model's Application Libraries folder and double-click the file `coil_shape_optimization_geom_sequence.mph`.

4 In the **Geometry** toolbar, click **Build All**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



The geometry should now look like that in Figure 1.

GLOBAL DEFINITIONS

Geometry Parameters

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, type `Geometry Parameters` in the **Label** text field.

Parameters 2

1 In the **Home** toolbar, click **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
f0	1[kHz]	1000 Hz	Frequency
lastTurns	0.5	0.5	Outer loop current factor
dmax	3[cm]	0.03 m	Maximum coil translation

ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

5 In the tree, select **Built-in>Copper**.

6 Click **Add to Component** in the window toolbar.

7 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

Copper (mat2)

1 In the **Settings** window for **Material**, locate the **Geometric Entity Selection** section.

2 From the **Selection** list, choose **Objects to Mirror**.

MAGNETIC FIELDS (MF)

Coil 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Magnetic Fields (mf)** and choose the domain setting **Coil**.

2 In the **Settings** window for **Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Inner Coils**.

4 Locate the **Coil** section. Select the **Coil group** check box.

5 In the I_{coil} text field, type `1[kA]`.

Coil 2

1 Right-click **Coil 1** and choose **Duplicate**.

2 In the **Settings** window for **Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Outer Coils 2**.

4 Locate the **Coil** section. In the I_{coil} text field, type `1[kA]*lastTurns`.

Setting a coil with a fraction of the current is a way to take into account of a fraction of a full turn in 2D a axisymmetric model; similarly a current with opposite sign may represent the same a coil that is wounded in the opposite direction.

Coil 3

1 Right-click **Coil 2** and choose **Duplicate**.

2 In the **Settings** window for **Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Outer Coils**.

4 Locate the **Coil** section. In the I_{coil} text field, type `-1[kA]*lastTurns`.

MESH 1

Mapped 1

1 In the **Mesh** toolbar, click **Mapped**.

2 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Infinite Domains**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 In the **Settings** window for **Distribution**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Infinite Domain Boundaries**.

Free Triangular 1

In the **Mesh** toolbar, click **Free Triangular**.

Size 1

1 Right-click **Free Triangular 1** and choose **Size**.

2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Deforming Domain**.

5 Locate the **Element Size** section. From the **Predefined** list, choose **Extremely fine**.

6 Click **Build All**.

COMPONENT 1 (COMP1)

Free Shape Domain 1

1 In the **Definitions** toolbar, click **Optimization** and choose **Shape Optimization>Free Shape Domain**.

2 In the **Settings** window for **Free Shape Domain**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Deforming Domain**.

Transformation 1

1 In the **Definitions** toolbar, click **Optimization** and choose **Shape Optimization>Transformation**.

2 In the **Settings** window for **Transformation**, locate the **Geometric Entity Selection** section.

3 From the **Selection** list, choose **Objects to Mirror**.

4 Locate the **Translation** section. In the table, enter the following settings:

	Lock	Lower bound (m)	Upper bound (m)
R		-dmax	dmax
Z		-coilSpace/4	coilSpace/4

5 Locate the **Scaling** section. From the **Scaling type** list, choose **No scaling**.

DEFINITIONS

Average inner magnetic field

1 In the **Definitions** toolbar, click **Probes** and choose **Domain Probe**.

2 In the **Settings** window for **Domain Probe**, type `Average inner magnetic field` in the **Label** text field.

3 In the **Variable name** text field, type `Bavg`.

4 Locate the **Source Selection** section. From the **Selection** list, choose **Rectangle 4**.

Average squared deviation from average

1 Right-click **Average inner magnetic field** and choose **Duplicate**.

2 In the **Settings** window for **Domain Probe**, type `Average squared deviation from average` in the **Label** text field.

3 In the **Variable name** text field, type `Bdev_sq`.

4 Locate the **Expression** section. In the **Expression** text field, type `(mf.normB-Bavg)^2`.

Average outer magnetic field

1 Right-click **Average squared deviation from average** and choose **Duplicate**.

2 In the **Settings** window for **Domain Probe**, type `Average outer magnetic field` in the **Label** text field.

3 In the **Variable name** text field, type `Bouter`.

4 Locate the **Source Selection** section. From the **Selection** list, choose **Outer Objective Domain**.

5 Locate the **Expression** section. In the **Expression** text field, type `mf.normB`.

Infinite Element Domain 1 (ie1)

1 In the **Definitions** toolbar, click **Infinite Element Domain**.

2 In the **Settings** window for **Infinite Element Domain**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Infinite Domains**.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `f0`.

Optimization

1 In the **Study** toolbar, click **Optimization** and choose **Optimization**.

2 In the **Settings** window for **Optimization**, locate the **Optimization Solver** section.

3 From the **Method** list, choose **MMA**.

4 Find the **Solver settings** subsection. From the **Keep solutions** list, choose **Every Nth**.

5 In the **Save every Nth** text field, type `1000`.

This effectively saves the first and last iteration.

6 Locate the **Objective Function** section. From the **Objective scaling** list, choose **Initial solution based**.

7 Click **Add Expression** in the upper-right corner of the **Objective Function** section. From the menu, choose **Component 1 (comp1)>Definitions>comp1.Bdev_sq - Average squared deviation from average - kg²/(s⁴·A²)**.

8 Click **Add Expression** in the upper-right corner of the **Objective Function** section. From the menu, choose **Component 1 (comp1)>Definitions>comp1.Bouter - Average outer magnetic field - T**.

Take the square root of the `Bdev_sq`, so that the two objectives get the same unit.

9 Locate the **Objective Function** section. In the table, enter the following settings:

Expression	Description	Evaluate for
`sqrt(comp1.Bdev_sq)`	`Domain Probe 2`	Frequency Domain

10 Locate the **Control Variables and Parameters** section. Click **Add**.

Initialize the study to generate a plot for use while solving.

11 In the table, enter the following settings:

Parameter name	Initial value	Scale	Lower bound	Upper bound
lastTurns (Outer loop current factor)	`0.5`	1	0	1

12 Locate the **Output While Solving** section. From the **Probes** list, choose **None**.

13 In the **Model Builder** window, click **Study 1**.

14 In the **Settings** window for **Study**, type `Shape Optimization` in the **Label** text field.

15 In the **Study** toolbar, click **Get Initial Value**.

RESULTS

Shape Optimization

The plot shows the displacement. Add an **Annotation** feature to also show the value of the `lastTurns` parameter.

Annotation 1

1 In the **Model Builder** window, right-click **Shape Optimization** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `eval(lastTurns)`.

4 Locate the **Coloring and Style** section. Clear the **Show point** check box.

5 From the **Background color** list, choose **Gray**.

SHAPE OPTIMIZATION

Optimization

1 In the **Model Builder** window, under **Shape Optimization** click **Optimization**.

2 In the **Settings** window for **Optimization**, locate the **Output While Solving** section.

3 Select the **Plot** check box.

4 From the **Plot group** list, choose **Shape Optimization**.

Enable move limits to reduce the risk of inverted elements.

Solver Configurations

In the **Model Builder** window, expand the **Shape Optimization>Solver Configurations** node.

Solution 1 (sol1)

1 In the **Model Builder** window, expand the **Shape Optimization>Solver Configurations>Solution 1 (sol1)** node, then click **Optimization Solver 1**.

2 In the **Settings** window for **Optimization Solver**, locate the **Optimization Solver** section.

3 Select the **Move limits** check box.

4 Select the **Maximum number of outer iterations** check box. In the associated text field, type `50`.

5 Click **Compute**.

RESULTS

Magnetic Flux Density Norm, Revolved Geometry (mf)

1 In the **Magnetic Flux Density Norm, Revolved Geometry (mf)** toolbar, click **Plot**.

2 Click the **Zoom Extents** button in the **Graphics** toolbar.

Field on Axis

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Field on Axis` in the **Label** text field.

3 Locate the **Data** section. From the **Optimization solution** list, choose **First**.

4 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

Line Graph 1

1 Right-click **Field on Axis** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Whole Axis**.

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `z`.

6 Click to expand the **Legends** section. Select the **Show legends** check box.

7 From the **Legends** list, choose **Manual**.

8 In the table, enter the following settings:

Legends
Initial Design

Line Graph 2

1 Right-click **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Shape Optimization/Solution 1 (sol1)**.

4 From the **Optimization solution** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Optimized Design

Line Graph 3

1 In the **Model Builder** window, right-click **Field on Axis** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Inner Axis**.

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `z`.

6 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

7 From the **Width** list, choose **3**.

Line Graph 4

1 Right-click **Line Graph 3** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Shape Optimization/Solution 1 (sol1)**.

4 From the **Optimization solution** list, choose **Last**.

5 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

Line Graph 3, Line Graph 4

1 In the **Model Builder** window, under **Results>Field on Axis**, Ctrl-click to select **Line Graph 3** and **Line Graph 4**.

2 Right-click and choose **Duplicate**.

Line Graph 5

1 In the **Settings** window for **Line Graph**, locate the **Selection** section.

2 From the **Selection** list, choose **Outer Axis**.

Line Graph 6

1 In the **Model Builder** window, click **Line Graph 6**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Outer Axis**.

4 In the **Field on Axis** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compute the value of the objective functions before and after optimization.

Evaluation Group 1

In the **Results** toolbar, click **Evaluation Group**.

Global Evaluation 1

1 Right-click **Evaluation Group 1** and choose **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)> Definitions>Bdev_sq - Average squared deviation from average -** $kg^2/(s^4 \cdot A^2)$.

3 Click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Definitions>Bouter - Average outer magnetic field - T**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
sqrt(Bdev_sq)	T	
Bouter	T	Average outer magnetic field

5 In the **Evaluation Group 1** toolbar, click **Evaluate**.

Compute the value of the corners, so that one can construct an interpolation function for 3D verification.

Point Displacements

1 In the **Results** toolbar, click **Evaluation Group**.

2 In the **Settings** window for **Evaluation Group**, type `Point Displacements` in the **Label** text field.

3 Locate the **Data** section. From the **Optimization solution** list, choose **Last**.

Point Evaluation 1

1 Right-click **Point Displacements** and choose **Point Evaluation**.

2 In the **Settings** window for **Point Evaluation**, locate the **Selection** section.

3 From the **Selection** list, choose **Lower left point**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`r`	`m`	`r-coordinate`
`z`	`m`	`z-coordinate`

5 In the **Point Displacements** toolbar, click **Evaluate**.

Geometry Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **AC/DC>Electromagnetic Fields>Magnetic Fields (mf)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Frequency Domain**.

6 Click **Done**.

GEOMETRY 1

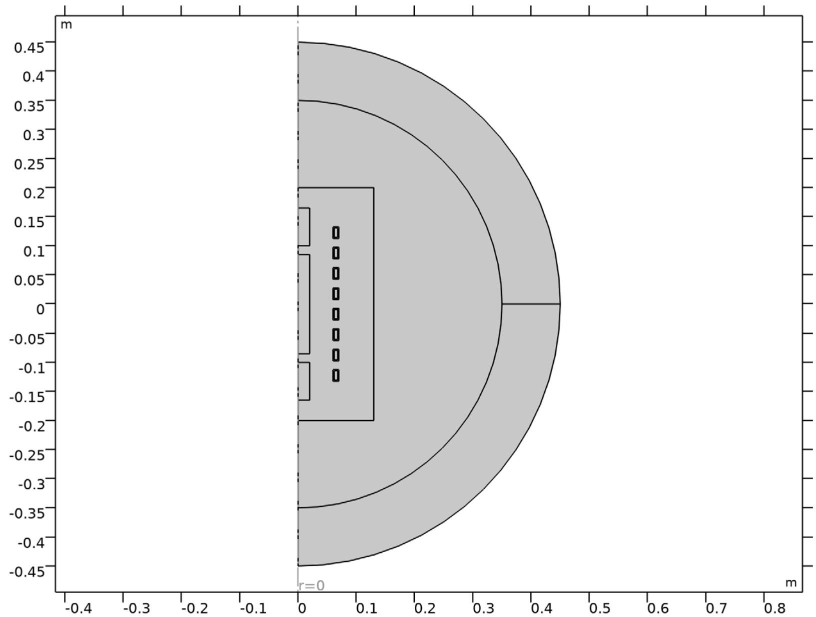
1 In the **Model Builder** window, expand the **Component 1 (comp1)>Geometry 1** node.

2 Right-click **Geometry 1** and choose **Insert Sequence**.

3 Browse to the model's Application Libraries folder and double-click the file `coil_shape_optimization_geom_sequence.mph`.

4 In the **Geometry** toolbar, click **Build All**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



The geometry should now look like that in Figure 1.

GLOBAL DEFINITIONS

Geometry Parameters

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, type `Geometry Parameters` in the **Label** text field.

Parameters 2

1 In the **Home** toolbar, click **Parameters** and choose **Add>Parameters**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
f0	1[kHz]	1000 Hz	Frequency
lastTurns	0.5	0.5	Outer loop current factor
dmax	3[cm]	0.03 m	Maximum coil translation

ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

5 In the tree, select **Built-in>Copper**.

6 Click **Add to Component** in the window toolbar.

7 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

Copper (mat2)

1 In the **Settings** window for **Material**, locate the **Geometric Entity Selection** section.

2 From the **Selection** list, choose **Objects to Mirror**.

MAGNETIC FIELDS (MF)

Coil 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Magnetic Fields (mf)** and choose the domain setting **Coil**.

2 In the **Settings** window for **Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Inner Coils**.

4 Locate the **Coil** section. Select the **Coil group** check box.

5 In the I_{coil} text field, type `1[kA]`.

Coil 2

1 Right-click **Coil 1** and choose **Duplicate**.

2 In the **Settings** window for **Coil**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Outer Coils 2**.

4 Locate the **Coil** section. In the I_{coil} text field, type `1[kA]*lastTurns`.

Setting a coil with a fraction of the current is a way to take into account of a fraction of a full turn in 2D a axisymmetric model; similarly a current with opposite sign may represent the same a coil that is wounded in the opposite direction.

Coil 3

1 Right-click **Coil 2** and choose **Duplicate**.
2 In the **Settings** window for **Coil**, locate the **Domain Selection** section.
3 From the **Selection** list, choose **Outer Coils**.
4 Locate the **Coil** section. In the I_{coil} text field, type `-1[kA]*lastTurns`.

MESH 1

Mapped 1

1 In the **Mesh** toolbar, click **Mapped**.
2 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.
3 From the **Geometric entity level** list, choose **Domain**.
4 From the **Selection** list, choose **Infinite Domains**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.
2 In the **Settings** window for **Distribution**, locate the **Boundary Selection** section.
3 From the **Selection** list, choose **Infinite Domain Boundaries**.

Free Triangular 1

In the **Mesh** toolbar, click **Free Triangular**.

Size 1

1 Right-click **Free Triangular 1** and choose **Size**.
2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.
3 From the **Geometric entity level** list, choose **Domain**.
4 From the **Selection** list, choose **Deforming Domain**.
5 Locate the **Element Size** section. From the **Predefined** list, choose **Extremely fine**.
6 Click **Build All**.

COMPONENT 1 (COMP1)

Free Shape Domain 1

1. In the **Definitions** toolbar, click **Optimization** and choose **Shape Optimization>Free Shape Domain**.
2. In the **Settings** window for **Free Shape Domain**, locate the **Domain Selection** section.
3. From the **Selection** list, choose **Deforming Domain**.

Transformation 1

1. In the **Definitions** toolbar, click **Optimization** and choose **Shape Optimization>Transformation**.
2. In the **Settings** window for **Transformation**, locate the **Geometric Entity Selection** section.
3. From the **Selection** list, choose **Objects to Mirror**.
4. Locate the **Translation** section. In the table, enter the following settings:

	Lock	Lower bound (m)	Upper bound (m)
R		`-dmax`	`dmax`
Z		`-coilSpace/4`	`coilSpace/4`

5. Locate the **Scaling** section. From the **Scaling type** list, choose **No scaling**.

DEFINITIONS

Average inner magnetic field

1. In the **Definitions** toolbar, click **Probes** and choose **Domain Probe**.
2. In the **Settings** window for **Domain Probe**, type `Average inner magnetic field` in the **Label** text field.
3. In the **Variable name** text field, type `Bavg`.
4. Locate the **Source Selection** section. From the **Selection** list, choose **Rectangle 4**.

Average squared deviation from average

1. Right-click **Average inner magnetic field** and choose **Duplicate**.
2. In the **Settings** window for **Domain Probe**, type `Average squared deviation from average` in the **Label** text field.
3. In the **Variable name** text field, type `Bdev_sq`.
4. Locate the **Expression** section. In the **Expression** text field, type `(mf.normB-Bavg)^2`.

Average outer magnetic field

1. Right-click **Average squared deviation from average** and choose **Duplicate**.

2 In the **Settings** window for **Domain Probe**, type `Average outer magnetic field` in the **Label** text field.

3 In the **Variable name** text field, type `Bouter`.

4 Locate the **Source Selection** section. From the **Selection** list, choose **Outer Objective Domain**.

5 Locate the **Expression** section. In the **Expression** text field, type `mf.normB`.

Infinite Element Domain 1 (ie1)

1 In the **Definitions** toolbar, click **Infinite Element Domain**.

2 In the **Settings** window for **Infinite Element Domain**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Infinite Domains**.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `f0`.

Optimization

1 In the **Study** toolbar, click **Optimization** and choose **Optimization**.

2 In the **Settings** window for **Optimization**, locate the **Optimization Solver** section.

3 From the **Method** list, choose **MMA**.

4 Find the **Solver settings** subsection. From the **Keep solutions** list, choose **Every Nth**.

5 In the **Save every Nth** text field, type `1000`.

This effectively saves the first and last iteration.

6 Locate the **Objective Function** section. From the **Objective scaling** list, choose **Initial solution based**.

7 Click **Add Expression** in the upper-right corner of the **Objective Function** section. From the menu, choose **Component 1 (comp1)>Definitions>comp1.Bdev_sq - Average squared deviation from average - kg^2/(s^4·A^2)**.

8 Click **Add Expression** in the upper-right corner of the **Objective Function** section. From the menu, choose **Component 1 (comp1)>Definitions>comp1.Bouter - Average outer magnetic field - T**.

Take the square root of the `Bdev_sq`, so that the two objectives get the same unit.

9 Locate the **Objective Function** section. In the table, enter the following settings:

Expression	Description	Evaluate for
`sqrt(comp1.Bdev_sq)`	`Domain Probe 2`	Frequency Domain

10 Locate the **Control Variables and Parameters** section. Click **Add**.

Initialize the study to generate a plot for use while solving.

11 In the table, enter the following settings:

Parameter name	Initial value	Scale	Lower bound	Upper bound
lastTurns (Outer loop current factor)	0.5	1	0	1

12 Locate the **Output While Solving** section. From the **Probes** list, choose **None**.

13 In the **Model Builder** window, click **Study 1**.

14 In the **Settings** window for **Study**, type `Shape Optimization` in the **Label** text field.

15 In the **Study** toolbar, click **Get Initial Value**.

RESULTS

Shape Optimization

The plot shows the displacement. Add an **Annotation** feature to also show the value of the `lastTurns` parameter.

Annotation 1

1 In the **Model Builder** window, right-click **Shape Optimization** and choose **Annotation**.

2 In the **Settings** window for **Annotation**, locate the **Annotation** section.

3 In the **Text** text field, type `eval(lastTurns)`.

4 Locate the **Coloring and Style** section. Clear the **Show point** check box.

5 From the **Background color** list, choose **Gray**.

SHAPE OPTIMIZATION

Optimization

1 In the **Model Builder** window, under **Shape Optimization** click **Optimization**.

2 In the **Settings** window for **Optimization**, locate the **Output While Solving** section.

3 Select the **Plot** check box.

4 From the **Plot group** list, choose **Shape Optimization**.

Enable move limits to reduce the risk of inverted elements.

Solver Configurations

In the **Model Builder** window, expand the **Shape Optimization>Solver Configurations** node.

Solution 1 (sol1)

1 In the **Model Builder** window, expand the **Shape Optimization>Solver Configurations> Solution 1 (sol1)** node, then click **Optimization Solver 1**.
2 In the **Settings** window for **Optimization Solver**, locate the **Optimization Solver** section.
3 Select the **Move limits** check box.
4 Select the **Maximum number of outer iterations** check box. In the associated text field, type `50`.
5 Click **Compute**.

RESULTS

Magnetic Flux Density Norm, Revolved Geometry (mf)

1 In the **Magnetic Flux Density Norm, Revolved Geometry (mf)** toolbar, click **Plot**.
2 Click the **Zoom Extents** button in the **Graphics** toolbar.

Field on Axis

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2 In the **Settings** window for **1D Plot Group**, type `Field on Axis` in the **Label** text field.
3 Locate the **Data** section. From the **Optimization solution** list, choose **First**.
4 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

Line Graph 1

1 Right-click **Field on Axis** and choose **Line Graph**.
2 In the **Settings** window for **Line Graph**, locate the **Selection** section.
3 From the **Selection** list, choose **Whole Axis**.
4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.
5 In the **Expression** text field, type `z`.
6 Click to expand the **Legends** section. Select the **Show legends** check box.
7 From the **Legends** list, choose **Manual**.

8 In the table, enter the following settings:

Legends
`Initial Design`

Line Graph 2

1 Right-click **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Shape Optimization/Solution 1 (sol1)**.

4 From the **Optimization solution** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
`Optimized Design`

Line Graph 3

1 In the **Model Builder** window, right-click **Field on Axis** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Inner Axis**.

4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `z`.

6 Click to expand the **Coloring and Style** section. From the **Color** list, choose **Cycle (reset)**.

7 From the **Width** list, choose **3**.

Line Graph 4

1 Right-click **Line Graph 3** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Shape Optimization/Solution 1 (sol1)**.

4 From the **Optimization solution** list, choose **Last**.

5 Locate the **Coloring and Style** section. From the **Color** list, choose **Cycle**.

Line Graph 3, Line Graph 4

1 In the **Model Builder** window, under **Results>Field on Axis**, Ctrl-click to select **Line Graph 3** and **Line Graph 4**.

2 Right-click and choose **Duplicate**.

Line Graph 5

1 In the **Settings** window for **Line Graph**, locate the **Selection** section.

2 From the **Selection** list, choose **Outer Axis**.

Line Graph 6

1 In the **Model Builder** window, click **Line Graph 6**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 From the **Selection** list, choose **Outer Axis**.

4 In the **Field on Axis** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compute the value of the objective functions before and after optimization.

Evaluation Group 1

In the **Results** toolbar, click **Evaluation Group**.

Global Evaluation 1

1 Right-click **Evaluation Group 1** and choose **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Definitions>Bdev_sq - Average squared deviation from average - kg²/(s⁴·A²)**.

3 Click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Definitions>Bouter - Average outer magnetic field - T**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`sqrt(Bdev_sq)`	`T`	
`Bouter`	`T`	`Average outer magnetic field`

5 In the **Evaluation Group 1** toolbar, click **Evaluate**.

Compute the value of the corners, so that one can construct an interpolation function for 3D verification.

Point Displacements

1 In the **Results** toolbar, click **Evaluation Group**.

2 In the **Settings** window for **Evaluation Group**, type `Point Displacements` in the **Label** text field.

3 Locate the **Data** section. From the **Optimization solution** list, choose **Last**.

Point Evaluation 1

1 Right-click **Point Displacements** and choose **Point Evaluation**.

2 In the **Settings** window for **Point Evaluation**, locate the **Selection** section.

3 From the **Selection** list, choose **Lower left point**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
r	m	r-coordinate
z	m	z-coordinate

5 In the **Point Displacements** toolbar, click **Evaluate**.