Computing Capacitance

Introduction

A capacitor, in its simplest form, is a two terminal electrical device that stores electric energy when a voltage is applied across the terminals. The stored electric energy is proportional to the applied voltage squared and is determined by the capacitance of the device. This example introduces a model of a simple capacitor. The electric field and device capacitance are solved for under electrostatic conditions.

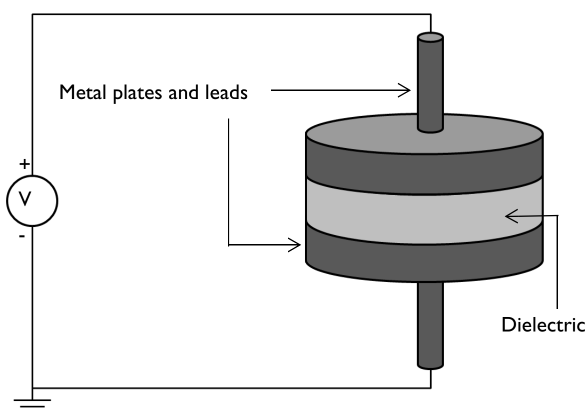


Figure 1: A simple capacitor composed of a disk of dielectric with metal plates on either side, and lead wires.

Model Definition

The capacitor being modeled is shown in Figure 1. Two metal disks, with leads, are separated by a disk of dielectric material. Since there can be significant fringing fields around the capacitor plates, an air volume is included in the model. The size of this air volume truncates the modeling space. In actuality, the fringing electric fields extend to infinity, but drop off in proportion to the inverse cube of the distance. They rapidly become small enough to be considered numerically insignificant. Here, it is assumed that the air volume is large enough to accurately capture the fringing fields. This could be checked by increasing the air volume size and comparing the results.

Under the assumption of electrostatic conditions, the entire surface of each electrode must be at the same potential, otherwise current would flow through these conductors. The air and dielectric are assumed to be perfect insulators. The quantity of interest is the electric potential in the air and in the dielectric — it is not necessary to solve for the potential in the electrode, because it is constant. There are two approaches that can be followed:

- apply the **Terminal** domain feature on the electrodes, or
- remove the electrode domains from the selection of the physics interface and apply appropriate boundary conditions (such as **Ground** or boundary **Terminal**) at the interface with the air or dielectric.

This model serves as an example of the second approach.

Results and Discussion

The electric field is plotted in Figure 2. The field is relatively uniform between the plates, but some strong variations are seen at the edges of the plates. The fringing fields are observed to extend a small distance away from the capacitor. The capacitance of the device is evaluated to be approximately 43 pF. In Figure 3, a slice plot of the electric potential shows that there is an equipotential surface exactly midway between the capacitor plates.

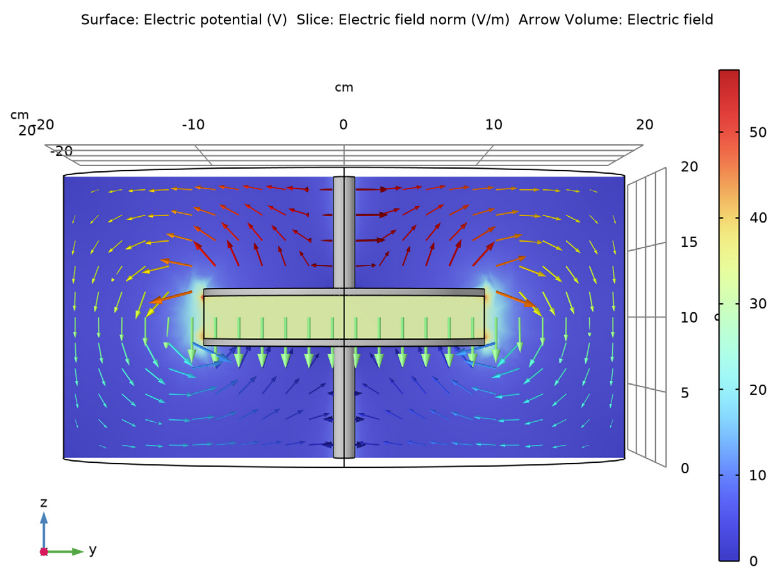


Figure 2: The electric field strength in the dielectric and air domain surrounding the capacitor.

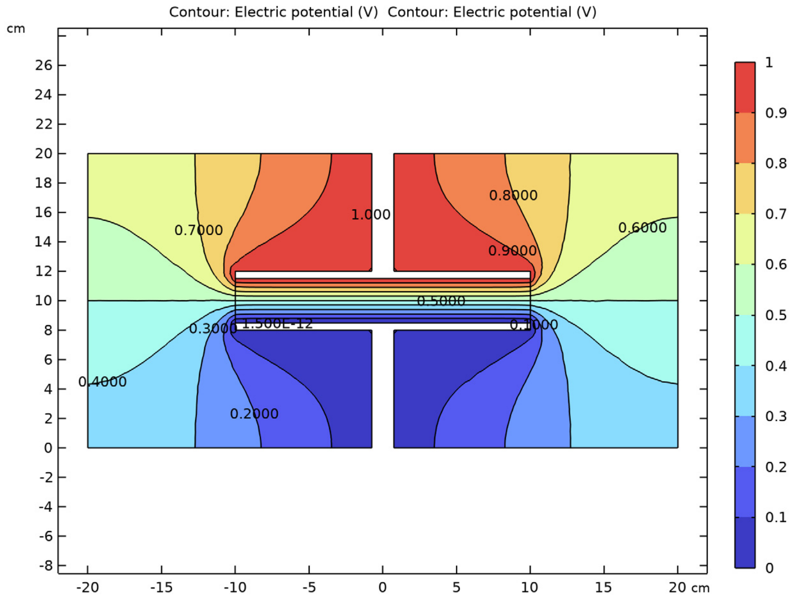


Figure 3: Contours of the electric potential on a slice through the center of the capacitor.

Application Library path: `ACDC_Module/Introductory_Electrostatics/capacitor_dc`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **AC/DC>Electric Fields and Currents>Electrostatics (es)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **cm**.

First, create a cylinder for the model domain.

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `20`.

4 In the **Height** text field, type `20`.

5 Click **Build Selected**.

Choose wireframe rendering to get a better view of the interior parts.

6 Click the **Wireframe Rendering** button in the **Graphics** toolbar.

Then, add a cylinder for the disc of dielectric with the two metal plates.

Cylinder 2 (cyl2)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `10`.

4 In the **Height** text field, type `4`.

5 Locate the **Position** section. In the **z** text field, type `8`.

6 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (cm)
`Layer 1`	`5[mm]`

7 Clear the **Layers on side** check box.

8 Select the **Layers on bottom** check box.

9 Select the **Layers on top** check box.

10 Click **Build Selected**.

Finish the geometry by adding two cylinders for the leads.

Cylinder 3 (cyl3)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `0.75`.

4 In the **Height** text field, type `8`.

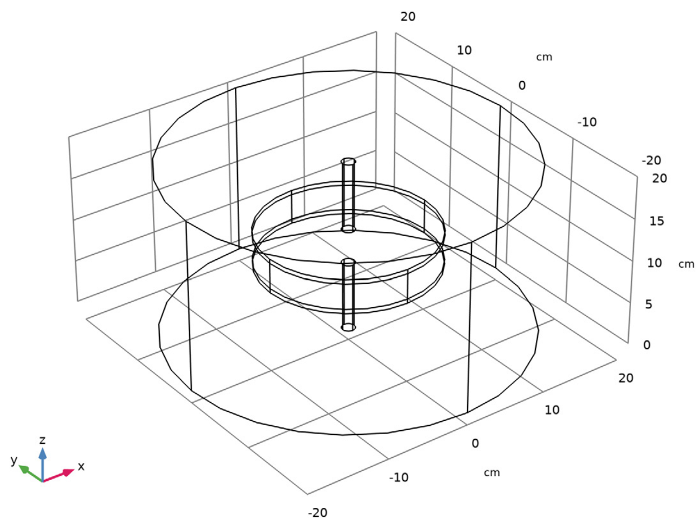
Cylinder 4 (cyl4)

1 Right-click **Cylinder 3 (cyl3)** and choose **Duplicate**.

2 In the **Settings** window for **Cylinder**, locate the **Position** section.

3 In the **z** text field, type `12`.

4 Click **Build All Objects**.



The result should look like the image above.

DEFINITIONS

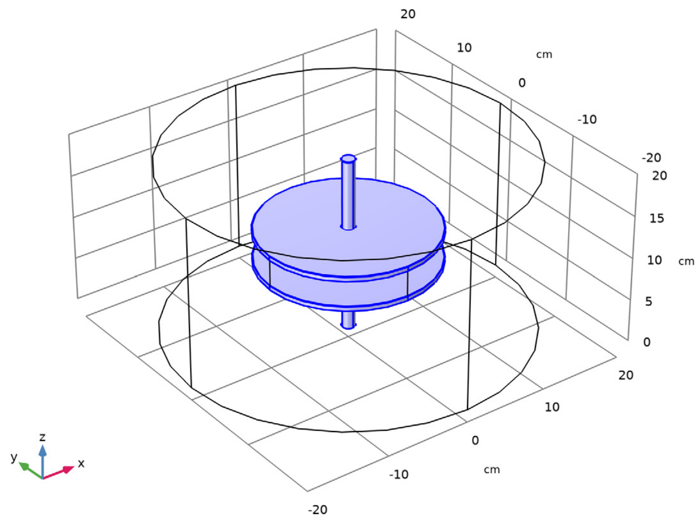
The model is composed of a disc of dielectric material with metal plates on either side and two lead wires. Create a set of selections to use when setting up the physics. First, create a selection for the metal domains.

Metal

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Metal` in the **Label** text field.

3 Select Domains 2 and 4–6 only.



Add a **Complement** selection for the insulators.

Insulators

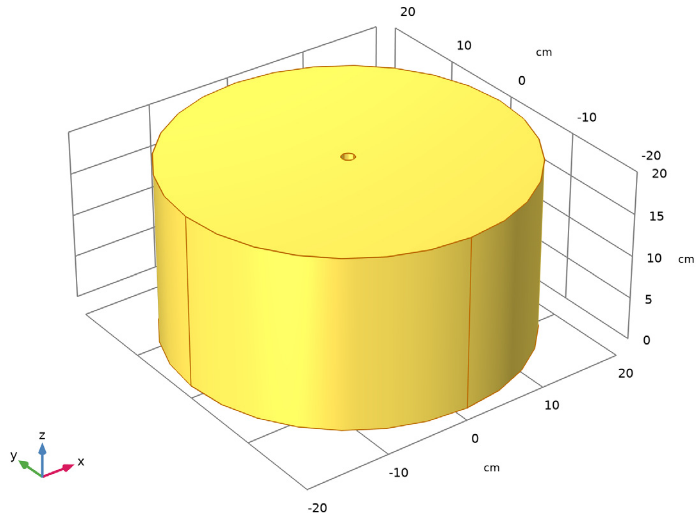
1 In the **Definitions** toolbar, click **Complement**.

2 In the **Settings** window for **Complement**, type `Insulators` in the **Label** text field.

3 Locate the **Input Entities** section. Under **Selections to invert**, click **Add**.

4 In the **Add** dialog box, select **Metal** in the **Selections to invert** list.

5 Click **OK**.



Add a selection for the ground boundaries, and the terminal boundaries.

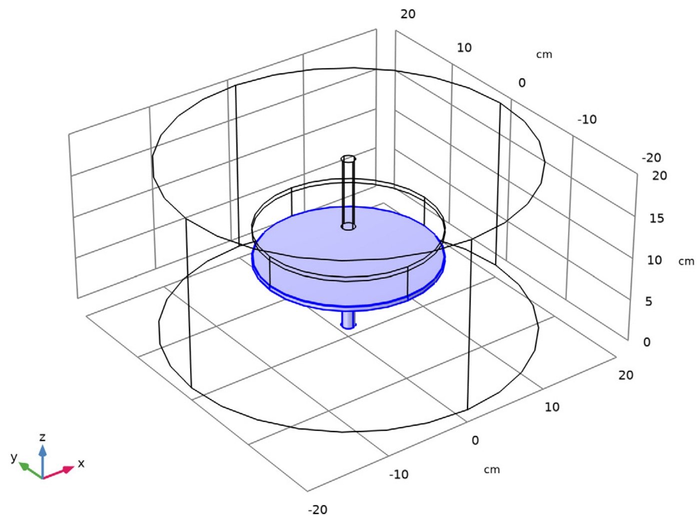
Ground

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Ground` in the **Label** text field.

3 Select Domains 2 and 5 only.

4 Locate the **Output Entities** section. From the **Output entities** list, choose **Adjacent boundaries**.



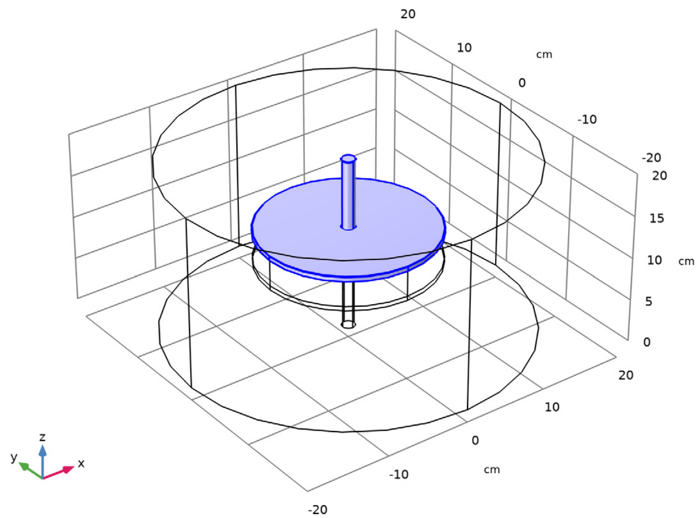
Terminal

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Terminal` in the **Label** text field.

3 Select Domains 4 and 6 only.

4 Locate the **Output Entities** section. From the **Output entities** list, choose **Adjacent boundaries**.



To get a better view, hide some of the boundaries. Begin by selecting the **Electrostatics** interface, then add a **Hide** node.

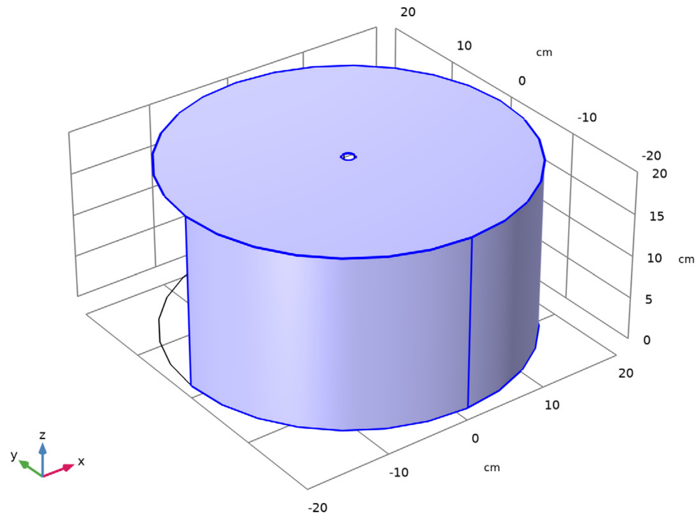
Hide for Physics 1

1 In the **Model Builder** window, right-click **View 1** and choose **Hide for Physics**.

2 In the **Settings** window for **Hide for Physics**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundaries 1, 4, and 23 only.



ELECTROSTATICS (ES)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electrostatics (es)**.

2 In the **Settings** window for **Electrostatics**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **Insulators**.

Ground 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Ground**.

2 In the **Settings** window for **Ground**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Ground**.

Terminal 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Terminal**.

2 In the **Settings** window for **Terminal**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Terminal**.

4 Locate the **Terminal** section. From the **Terminal type** list, choose **Voltage**.

Next, assign material properties to the model. Begin by specifying **Air** for all domains.

ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

Override the dielectric disc with glass (quartz).

5 In the tree, select **Built-in>Glass (quartz)**.

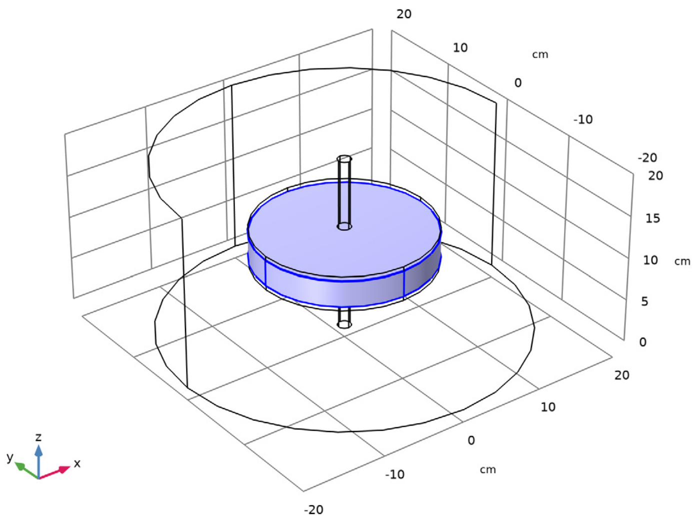
6 Click **Add to Component** in the window toolbar.

7 In the **Home** toolbar, click  **Add Material** to close the **Add Material** window.

MATERIALS

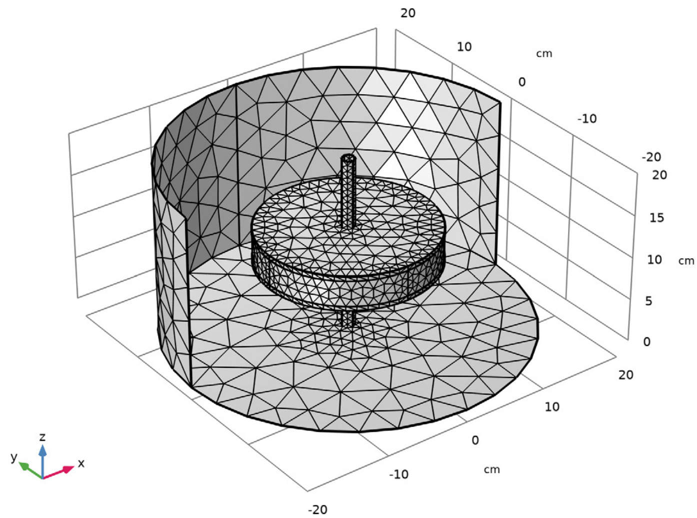
Glass (quartz) (mat2)

Select Domain 3 only.



MESH 1

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Build All**.



STUDY 1

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

4 In the **Home** toolbar, click **Compute**.

Add a solution and define a **Selection** for the metal parts. Domains that are excluded from the selection will be hidden in the corresponding plots.

RESULTS

Study 1/Solution 1 (2) (sol1)

1 In the **Model Builder** window, expand the **Results** node.

2 Right-click **Results>Datasets** and choose **Solution**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 From the **Selection** list, choose **Metal**.

3D Plot Group 1

In the **Results** toolbar, click **3D Plot Group**.

The metal parts can be visualized by choosing **Uniform** for the coloring type. In this case the purpose of the plot is not to show a quantity, but to show a shape. The variable that the plot is based on is of no importance. Create a surface plot for this purpose.

Surface 1

1 Right-click **3D Plot Group 1** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Solution 1 (2) (sol1)**.

4 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.

5 From the **Color** list, choose **Gray**.

3D Plot Group 1

Add a slice plot for the norm of the electric fields and an arrow plot for the electric fields.

Slice 1

1 In the **Model Builder** window, right-click **3D Plot Group 1** and choose **Slice**.

2 In the **Settings** window for **Slice**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Electrostatics>Electric>es.normE - Electric field norm - V/m**.

3 Locate the **Plane Data** section. In the **Planes** text field, type 1.

4 Locate the **Coloring and Style** section. Click **Change Color Table**.

5 In the **Color Table** dialog box, select **Rainbow>RainbowLight** in the tree.

6 Click **OK**.

Arrow Volume 1

1 Right-click **3D Plot Group 1** and choose **Arrow Volume**.

2 In the **Settings** window for **Arrow Volume**, locate the **Arrow Positioning** section.

3 Find the **x grid points** subsection. In the **Points** text field, type 1.

4 Find the **y grid points** subsection. In the **Points** text field, type 24.

5 Find the **z grid points** subsection. In the **Points** text field, type 11.

6 Locate the **Coloring and Style** section. From the **Arrow length** list, choose **Logarithmic**.

Color Expression 1

1 Right-click **Arrow Volume 1** and choose **Color Expression**.

2 In the **Settings** window for **Color Expression**, locate the **Coloring and Style** section.

3 Clear the **Color legend** check box.

4 Click the **Go to YZ View** button in the **Graphics** toolbar.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

The plot should look like the one in Figure 2.

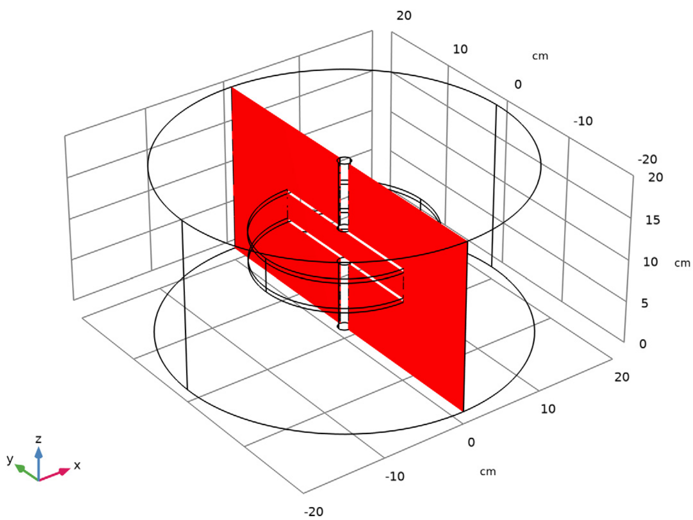
Create a **Cut Plane** that may serve as a basis for 2D plots.

Cut Plane 1

1 In the **Results** toolbar, click **Cut Plane**.

2 In the **Settings** window for **Cut Plane**, click **Plot**.

3 Click the **Go to Default View** button in the **Graphics** toolbar.



On this dataset, base a **2D Plot Group** with contour plots for the electric potential.

2D Plot Group 2

In the **Results** toolbar, click **2D Plot Group**.

Contour 1

1 Right-click **2D Plot Group 2** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Levels** section.

3 From the **Entry method** list, choose **Levels**.

4 Click **Range**.

5 In the **Range** dialog box, type `0.1` in the **Start** text field.

6 In the **Step** text field, type `0.1`.

7 In the **Stop** text field, type `0.9`.

8 Click **Replace**.

9 In the **Settings** window for **Contour**, locate the **Coloring and Style** section.

10 From the **Contour type** list, choose **Filled**.

11 Click **Change Color Table**.

12 In the **Color Table** dialog box, select **Rainbow>RainbowLight** in the tree.

13 Click **OK**.

Contour 2

1 In the **Model Builder** window, right-click **2D Plot Group 2** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Levels** section.

3 From the **Entry method** list, choose **Levels**.

4 Click **Range**.

5 In the **Range** dialog box, type `0` in the **Start** text field.

6 In the **Step** text field, type `0.1`.

7 In the **Stop** text field, type `1`.

8 Click **Replace**.

9 In the **Settings** window for **Contour**, locate the **Coloring and Style** section.

10 Select the **Level labels** check box.

11 From the **Coloring** list, choose **Uniform**.

12 From the **Color** list, choose **Black**.

13 Clear the **Color legend** check box.

14 In the **2D Plot Group 2** toolbar, click **Plot**.

15 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the resulting plot with Figure 3.

The **Terminal** feature automatically computes the capacitance. The computed value is available in a global variable and can be accessed using a **Global Evaluation**.

Global Evaluation 1

1 In the **Results** toolbar, click 8.5 **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Electrostatics>Terminals>es.C11 - Maxwell capacitance - F**.

3 Click **Evaluate**.

TABLE

1 Go to the **Table** window.

The computed capacitance should be around 43 pF.