

# Axisymmetric Approximation of 3D Inductor

# Introduction

When the frequency is high enough, capacitative effects can become important also for devices that are inductive and/or resistive at lower frequencies. Modeling this effect in an inductor requires accounting for electric field components both parallel and perpendicular to the wire. This easily leads to the conclusion that a 3D model is always necessary even if the coil is a low slant helix. This is not always the case, as shown in this tutorial.

Starting from the 3D inductor model described in the *Introduction to AC/DC Module* manual, a 2D axisymmetric model able to describe a self resonating inductor is created. In order to build an equivalent 2D axisymmetric model, an effective axisymmetric core is drawn and the RLC Coil Group feature is used.

The method shown here is particularly suitable for studying systems with thousands of turns, such as sensors or transformers, with limited computational power.

# Model Definition

The 3D solution of the power inductor of Figure 1 is presented in the manual *Introduction to AC/DC Module*. We are considering the same system.



Figure 1: The 3D geometry representation of the power inductor under study.

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In that device, the coil can to a reasonable level of approximation be treated as axisymmetric. Therefore, any of its cross-sections are suitable for performing a 2D axisymmetric analysis from a geometric point of view.

On the other hand, the core geometry shape indicates that a 3D analysis is required. However, in order to have a description which is correct from a magnetic point of view, the core does not have to be rotationally symmetric as long as:

- it is not supporting 3D non axisymmetric eddy currents
- an equivalent axisymmetric geometry respecting the original core reluctance is used

Such a construction, i.e replacing the 3D core with a 2D axisymmetric representation that preserves the crucial area that influences the magnetic circuit, is shown in Figure 2.



Figure 2: The 2D axisymmetric geometry equivalent to the power inductor under study.

The parameters are defined in Table 1. The original external column area (as given by the *Outer column section area* variable in Table 1) is the area represented in Figure 3. It is exactly  $\pi(r_{out}^2 - r_{in}^2)$  in the 2D axisymmetric geometry. A similar consideration is valid for the *Upper magnetic circuit closure*.

The coil geometry representation in 2D axial symmetry is directly taken from the midplane cross section on the *xz*-plane in 3D. Figure 4 shows the final 2D axisymmetric geometry.

	3D	2D axial symmetry	2D axisymmetric variable name
Inner column radius	3.5[cm]	3.5[cm]	-
Outer column internal radius	6.5[cm]	6.5[cm]	r_in
Outer column section area	19.92[cm^2]	identical by construction	external_area
Outer column external radius	-	sqrt(external_area/ pi+r_in^2)	r_out
Upper magnetic circuit closure area	7.5[cm^2]	identical by construction	upper_area
Upper magnetic circuit closure	-	upper_area/pi/r_in	h_eq

TABLE I: QUOTA OF THE 3D AND THE 2D AXISYMMETRIC EQUIVALENT.

The main challenge of the 2D axisymmetric coil model is to account for the way that the electric currents will flow. The RLC Coil Group feature available in the Magnetic and Electric Fields interface automatically takes into account that the currents are balanced among:

- I conduction currents and induced currents that are flowing in the azimuthal direction
- 2 displacement currents that flow in the *rz*-plane from one turn to the other

In the low frequency limit, currents are all of the first type. In the high frequency limit, the currents are all of the second. At some intermediate frequency there is a resonant frequency where inductive and capacitive effects perfectly balance and the coil self-resonates. At this frequency the inductor is purely resistive and the total loss peaks as a function of frequency.

In the present model the core is not grounded and there is no other external electric ground. Compared to a real system where the inductor often is mounted on or near a ground plane, this may cause a shift in the resonance frequency. This is easily added, e.g. by applying one or more electric boundary conditions to the core boundaries.



Figure 3: The 3D cross sectional area of the outer columns used to compute the 2D axisymmetric outer columns width.



Figure 4: The 2D axisymmetric geometry including some surrounding air.

# Results and Discussion

The field and loss plots can be compared to the 3D model discussed in the *Introduction to AC/DC Module* manual.



Figure 5: Magnetic flux density in the revolved 2D geometry.



Figure 6: The eddy current losses are shown (log scale with zero offset).

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Figure 7: Electric field between turns far from the resonance frequency.



Figure 8: Electric field between turns at the resonance frequency.

The most striking sign of the resonance is shown in Figure 9, where the real and imaginary parts of the coil impedance are shown. The resistance (blue curve) is shown in tens of  $k\Omega$  together with the reactance divided by the angular frequency (green curve). The green curve starts at low frequencies from the static inductance, grows, and then becomes negative. When the green curve becomes negative, it means that the coil behaves like a capacitor. The real part of the impedance accounts for losses, and peaks at the frequency where the capacitative and the inductive contributions to the overall system impedance balance out. The position and value of the peak featured by the 2D axisymmetric model also match the ones of the original 3D model in the *Introduction to AC/DC Module* manual fairly well.



Figure 9: The inductor impedance as a function of frequency. Compare this plot to the corresponding plot of the 3D Inductor in the Introduction to AC/DC Module manual.

**Application Library path:** ACDC\_Module/Devices,\_Inductive/ axisymmetric\_approximation\_of\_inductor\_3d

# Modeling Instructions

From the File menu, choose New.

## NEW

In the New window, click 🕙 Model Wizard.

# MODEL WIZARD

- I In the Model Wizard window, click 🚈 2D Axisymmetric.
- 2 In the Select Physics tree, select AC/DC>Electromagnetic Fields>Vector Formulations> Magnetic and Electric Fields (mef).
- 3 Click Add.
- 4 In the Added physics interfaces tree, select Magnetic and Electric Fields (mef).
- 5 Click 🔿 Study.
- 6 In the Select Study tree, select General Studies>Frequency Domain.
- 7 Click **M** Done.

First import the 3D geometry and take some measurements to generate the axisymmetric equivalent.

# PART I

In the Model Builder window, right-click Global Definitions and choose Geometry Parts> 3D Part.

Import I (imp1)

- I In the Home toolbar, click া Import.
- 2 In the Settings window for Import, locate the Import section.
- 3 Click 📂 Browse.
- 4 Browse to the model's Application Libraries folder and double-click the file inductor\_3d.mphbin.

5 Click 🔚 Import.



Work Plane I (wp1)

- I In the Geometry toolbar, click 📥 Work Plane.
- 2 In the Settings window for Work Plane, locate the Plane Definition section.
- 3 From the Plane list, choose xz-plane.
- 4 Click 📥 Show Work Plane.

Work Plane 1 (wp1)>Cross Section 1 (cro1)

- I In the Work Plane toolbar, click 🔶 Cross Section.
- 2 In the Settings window for Cross Section, click 틤 Build Selected.

Work Plane I (wp1)

- I In the Model Builder window, under Global Definitions>Geometry Parts>Part I click Work Plane I (wp1).
- 2 In the Settings window for Work Plane, click 틤 Build Selected.

Delete Entities I (dell)

- I In the Model Builder window, right-click Part I and choose Delete Entities.
- 2 In the Settings window for Delete Entities, locate the Entities or Objects to Delete section.
- 3 From the Geometric entity level list, choose Domain.

- 4 On the object impl, select Domains 2–5 only.
- **5** Click the  $4 \rightarrow$  **Zoom Extents** button in the **Graphics** toolbar.



6 Click 틤 Build Selected.

Work Plane 2 (wp2)

- I In the Geometry toolbar, click <del>§</del> Work Plane.
- 2 In the Settings window for Work Plane, locate the Plane Definition section.
- 3 In the z-coordinate text field, type 5[mm].
- 4 Click 📥 Show Work Plane.

Work Plane 2 (wp2)>Cross Section 1 (cro1)

- I In the Work Plane toolbar, click 🔶 Cross Section.
- 2 In the Settings window for Cross Section, click 틤 Build Selected.

Work Plane 2 (wp2)

- I In the Model Builder window, under Global Definitions>Geometry Parts>Part I click Work Plane 2 (wp2).
- 2 In the Settings window for Work Plane, click 틤 Build Selected.

Work Plane 3 (wp3)

- I In the Geometry toolbar, click 📥 Work Plane.
- 2 In the Settings window for Work Plane, locate the Plane Definition section.

- 3 From the Plane type list, choose Face parallel.
- 4 On the object dell, select Boundary 46 only.



5 Click 📥 Show Work Plane.

Work Plane 3 (wp3)>Cross Section 1 (cro1)

- I In the Work Plane toolbar, click 🔶 Cross Section.
- 2 In the Settings window for Cross Section, click 🔚 Build Selected.

# Work Plane 3 (wp3)

- In the Model Builder window, under Global Definitions>Geometry Parts>Part I click Work Plane 3 (wp3).
- 2 In the Settings window for Work Plane, click 틤 Build Selected.

The next step returns in the message log the relevant quota for the 3D to 2Daxi reduction.

- **3** Click the 🗮 Wireframe Rendering button in the Graphics toolbar.
- 4 In the Home toolbar, click 📗 Build All.

Next, make some measurements for use when creating the axisymmetric geometry.

- 5 In the Model Builder window, click Part I.
- 6 In the Graphics window toolbar, click ▼ next to 📄 Select Boundaries, then choose Select Points.

7 On the object wp2, select Points 9 and 14 only.

It might be easier to select the points by using the **Selection List** window. To open this window, in the **Home** toolbar click **Windows** and choose **Selection List**. (If you are running the cross-platform desktop, you find **Windows** in the main menu.)



- 8 In the Geometry toolbar, click Measure.Note the distance value, 0.07 m, and that the points have the same *y* and *z* coordinates.
- 9 On the object wp2, select Point 7 only.

**IO** On the object **wp3**, select Point 8 only.



II In the **Geometry** toolbar, click **Measure**.

The distance between the points is 0.13 m, again along an axis parallel to the *x*-axis.

12 In the Graphics window toolbar, click ▼ next to 💮 Select Points, then choose Select Boundaries.

**I3** On the object **wp2**, select Boundaries 1, 2, 5, and 6 only.



**I4** In the **Geometry** toolbar, click **Measure**.

**I5** On the object **wp3**, select Boundaries 1 and 5 only.



**I6** In the **Geometry** toolbar, click **Measure**.

The combined area of the two faces is  $7.5 \cdot 10^{-4} \text{ m}^2$ . You will later use this value when drawing the 3D area of the upper magnetic circuit closure of the axisymmetric equivalent core.

# GLOBAL DEFINITIONS

Add parameters for drawing the axisymmetric equivalent core.

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
inner_diameter	0.07[m]	0.07 m	3D diameter of central column
outer_diameter	0.13[m]	0.13 m	3D inner distance between external columns
external_area	0.001992 [m^2]	0.001992 m <sup>2</sup>	3D area of one of the two lateral columns
upper_area	7.5e-4[m^2]	7.5E-4 m <sup>2</sup>	3D area of the upper magnetic circuit closure
r_in	outer_diameter/2	0.065 m	2Daxi equivalence for external radius of lateral column
r_out	sqrt(external_area /pi+r_in^2)	0.069707 m	2Daxi equivalence for external radius of lateral column
h_eq	upper_area/pi/r_in	0.0036728 m	2Daxi equivalence for height of upper magnetic circuit closure

# GEOMETRY I

Import I (imp1)

- I In the **Home** toolbar, click া Import.
- 2 In the Settings window for Import, locate the Import section.

- **3** From the **Source** list, choose **Geometry sequence**.
- 4 From the Geometry list, choose Work Plane I (wpl), Part I.
- 5 Click Import.

# Rectangle 1 (r1)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- **3** In the **Width** text field, type r\_out.
- 4 In the Height text field, type 0.0325+2\*h\_eq.
- 5 Locate the Position section. In the z text field, type -0.004-h\_eq.

## Circle I (c1)

- I In the **Geometry** toolbar, click  $\bigcirc$  **Circle**.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type 0.1.
- 4 Locate the **Position** section. In the **z** text field, type 0.01.
- 5 Locate the Size and Shape section. In the Sector angle text field, type 180.
- 6 Locate the Rotation Angle section. In the Rotation text field, type -90.

## Intersection 1 (int1)

- I In the Geometry toolbar, click pooleans and Partitions and choose Intersection.
- 2 Select the objects impl and rl only.

## Form Union (fin)

I In the Geometry toolbar, click 📗 Build All.



# MATERIALS

Add material data for the air and for the copper windings.

# Material I (mat1)

- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, locate the Material Contents section.
- **3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic

Property	Variable	Value	Unit	Property group
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	I	Basic

Add the constitutive relationship and the material information for the lossy iron core.

Material 2 (mat2)

- I Right-click Materials and choose Blank Material.
- 2 Select Domain 2 only.



Material 3 (mat3)

I Right-click Materials and choose Blank Material.

**2** Select Domains 4–6 only.





**4** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	I	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	6e7	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	I	Basic

# MAGNETIC AND ELECTRIC FIELDS (MEF)

Ampère's Law and Current Conservation 2

In the Model Builder window, under Component I (compl) right-click
Magnetic and Electric Fields (mef) and choose Ampère's Law and Current Conservation.

- **2** Select Domain 2 only.
- **3** In the Settings window for Ampère's Law and Current Conservation, locate the Constitutive Relation B-H section.
- 4 From the Magnetization model list, choose Magnetic losses.

Magnetic Insulation 1

In the Model Builder window, click Magnetic Insulation I.

Electric Insulation 1

- I In the Physics toolbar, click Attributes and choose Electric Insulation.
- 2 Click the **Example 2** Select All button in the Graphics toolbar.

# MATERIALS

Material 2 (mat2)

- I In the Model Builder window, under Component I (compl)>Materials click Material 2 (mat2).
- 2 In the Settings window for Material, locate the Material Contents section.
- **3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability (real part)	murPrim	1200	I	Magnetic losses
Relative permeability (imaginary part)	murBis	100	I	Magnetic losses
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

## MAGNETIC AND ELECTRIC FIELDS (MEF)

First add the Ampere's law, which is necessary for adding the RLC Coil Feature.

Ampère's Law 1

I In the Physics toolbar, click 🔵 Domains and choose Ampère's Law.

**2** Select Domains 4–6 only.

Add the RLC Coil Feature, which takes care of coupling the transverse currents inside the conductors to the capacitative current flowing perpendicularly to the windings.

RLC Coil Group 1

- I In the Physics toolbar, click **Domains** and choose **RLC Coil Group**.
- **2** Select Domains 4–6 only.

As the coils are connected from the bottom to the top and domain ordering is not the same, a manual ordering is necessary.

- 3 In the Settings window for RLC Coil Group, locate the Geometry section.
- 4 From the Domain ordering list, choose Manual.
- 5 Locate the Domain Selection section. In the list, select 5.
- 6 Locate the Geometry section. In the Domain list text field, type 5, 4, 6.

In order to improve variable scaling, set a ground voltage slightly different from zero.

7 Locate the **RLC Coil Group** section. In the  $V_0$  text field, type 1[mV].

# MESH I

Free Triangular 1

- I In the Mesh toolbar, click Kree Triangular.
- 2 In the Settings window for Free Triangular, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.

4 Select Domains 2–6 only.



## Size 1

I Right-click Free Triangular I and choose Size.

- **2** Select Domains 2 and 3 only.
- 3 In the Settings window for Size, locate the Element Size section.
- 4 Click the **Custom** button.
- 5 Locate the Element Size Parameters section.
- 6 Select the Maximum element size check box. In the associated text field, type 2e-3.

## Size 2

- I In the Model Builder window, right-click Free Triangular I and choose Size.
- 2 In the Settings window for Size, locate the Geometric Entity Selection section.
- 3 Click Clear Selection.
- **4** Select Domains 4–6 only.
- 5 Locate the **Element Size** section. Click the **Custom** button.
- 6 Locate the Element Size Parameters section.
- 7 Select the Maximum element size check box. In the associated text field, type 5e-4.

# Free Triangular 2

I In the Mesh toolbar, click Kree Triangular.

2 In the Settings window for Free Triangular, click 📗 Build Selected.

Boundary Layers 1

- I In the Mesh toolbar, click Boundary Layers.
- 2 In the Settings window for Boundary Layers, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domains 4–6 only.

A boundary layer mesh is added in order to resolve the skin depth.

Boundary Layer Properties

- I In the Model Builder window, click Boundary Layer Properties.
- **2** In the **Settings** window for **Boundary Layer Properties**, locate the **Boundary Selection** section.
- 3 From the Selection list, choose All boundaries.
- **4** Locate the Layers section. From the Thickness specification list, choose First layer.
- 5 In the Number of layers text field, type 12.
- 6 In the Stretching factor text field, type 1.3.
- 7 In the Thickness text field, type 10[um].
- 8 Click 🖷 Build Selected.

## STUDY I

Step 1: Frequency Domain

- I In the Model Builder window, under Study I click Step I: Frequency Domain.
- 2 In the Settings window for Frequency Domain, locate the Study Settings section.
- 3 From the Frequency unit list, choose MHz.
- 4 In the **Frequencies** text field, type range(1,0.25,10).
- **5** In the **Home** toolbar, click **= Compute**.

## RESULTS

Magnetic Flux Density Norm (mef)

Click the **Zoom Extents** button in the **Graphics** toolbar.

Study I/Solution I (soll)

In the Model Builder window, expand the Results>Datasets node, then click Study I/ Solution I (soll).

## Selection

- I In the Results toolbar, click 🗞 Attributes and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Domain.
- 4 Select Domains 2 and 4–6 only.

Magnetic Flux Density Norm, Revolved Geometry (mef)



# Study I/Solution I (2) (soll)

In the **Results** toolbar, click **More Datasets** and choose **Solution**.

# Selection

- I In the Results toolbar, click 🐐 Attributes and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- **3** From the **Geometric entity level** list, choose **Domain**.
- **4** Select Domains 3–6 only.

# Resistive Losses

- I In the **Results** toolbar, click **2D Plot Group**.
- 2 In the Settings window for 2D Plot Group, type Resistive Losses in the Label text field.

- 3 Locate the Data section. From the Dataset list, choose Study I/Solution I (2) (soll).
- 4 From the Parameter value (freq (MHz)) list, choose I.

## Surface 1

- I Right-click Resistive Losses and choose Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type log(mef.Qrh+0.1).
- **4** In the **Resistive Losses** toolbar, click **O Plot**.
- **5** Click the **Com Extents** button in the **Graphics** toolbar.



## Electric Field

- I In the Home toolbar, click 📠 Add Plot Group and choose 2D Plot Group.
- 2 In the Settings window for 2D Plot Group, type Electric Field in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study I/Solution I (2) (soll).

#### Arrow Surface 1

- I Right-click Electric Field and choose Arrow Surface.
- **2** Click the  $\longleftrightarrow$  **Zoom Extents** button in the **Graphics** toolbar.
- 3 In the Settings window for Arrow Surface, locate the Arrow Positioning section.

- 4 Find the r grid points subsection. In the Points text field, type 20.
- 5 Find the z grid points subsection. In the Points text field, type 20.
- 6 Click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Magnetic and Electric Fields>Electric>mef.Er, mef.Ez Electric field.

Color Expression I

- I Right-click Arrow Surface I and choose Color Expression.
- 2 In the Electric Field toolbar, click **O** Plot.
- **3** Click the **- Zoom Extents** button in the **Graphics** toolbar.



#### Electric Field

- I In the Model Builder window, under Results click Electric Field.
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Parameter value (freq (MHz)) list, choose 6.25.



**4** In the **Electric Field** toolbar, click **I** Plot.

Impedance

- I In the Home toolbar, click 📠 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Impedance in the Label text field.
- 3 Click to expand the Title section. From the Title type list, choose None.
- 4 Locate the Legend section. From the Position list, choose Upper left.
- 5 Locate the Plot Settings section. Select the Two y-axes check box.

Generate a plot with the real and the imaginary part of the impedance, which can be read at low frequency as impedance and resistance. The resonant peak is clearly visible and compares well with the original 3D geometry.

Global I

Right-click Impedance and choose Global.

Global 2

In the Model Builder window, right-click Impedance and choose Global.

Global I

I In the Settings window for Global, locate the y-Axis Data section.

**2** In the table, enter the following settings:

Expression	Unit	Description
<pre>real(mef.VCoil_1/1[A])</pre>	Ω	Real part of impedance

Global 2

- I In the Model Builder window, click Global 2.
- 2 In the Settings window for Global, locate the y-Axis section.
- 3 Select the Plot on secondary y-axis check box.
- 4 Locate the y-Axis Data section. In the table, enter the following settings:

Expression	Unit	Description
<pre>imag(mef.VCoil_1/1[A])</pre>	Ω	Imaginary part of impedance

**5** In the **Impedance** toolbar, click **O Plot**.



6 Click the 4 Zoom Extents button in the Graphics toolbar.